CROCO

Coastal and Regional Ocean COmmunity model

Patrick Marchesiello

Advances in nonhydrostatic CROCO

… towards realistic LES models for the ocean

Advanced Ocean Modeling with CROCO

January 19-21 2022

CROCO Summer school 2022 in Chile

Mesoscale | Submesoscale | Internal waves | Stratified Turbulence | Inertial Zone | Dissipative Zone

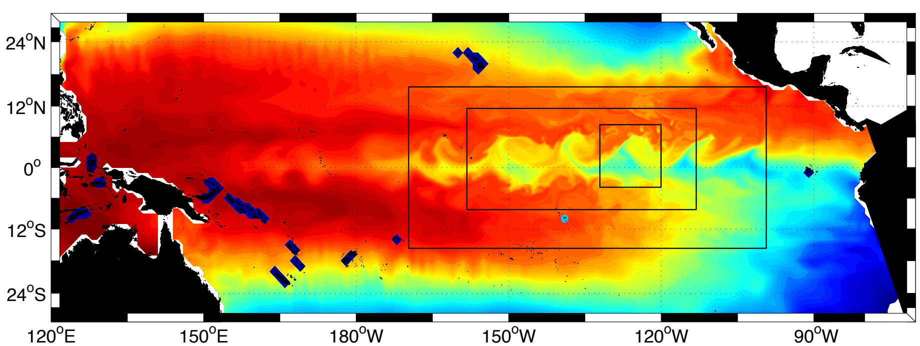
100km | 10km | 1km | 100m | 10m | 1m | 1cm | 1mm

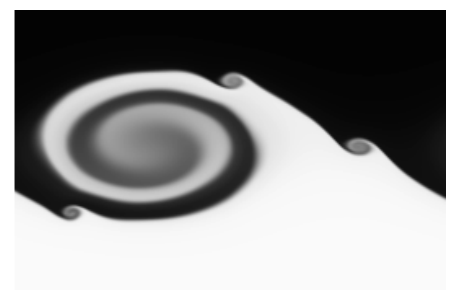
QG zone

Submesoscales

non-hydrostatic regime (RANS zone)

LES zone





Non-hydrostatic solver

- Pressure correction method (incompressible)
- Compressible approach

Non-hydrostatic solver: algorithm

- Pressure correction method (incompressible)
  - Roullet, Molemaker, Ducousso (LOPS-UCLA)

Non-hydrostatic solver: algorithm

- Pressure correction method

$$p = p_a + p_H + q$$

Homogeneous linearized equations

$$\begin{aligned}
\partial_x u &+ \partial_z w = 0 \\
\partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w &= -\partial_z q/\rho_0 \\
\\
\partial_t \eta &= w(0) = -H\partial_x \overline{u} \\
w|_{z=-H} &= 0 \\
q\,|_{z=0} &= 0
\end{aligned}$$

Non-hydrostatic solver: algorithm

- Pressure correction method

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w|_{z=-H} &= 0 \\
q|_{z=0} &= 0
\end{aligned}$$

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

$\partial_x u + \partial_z w = 0$

Correct velocity to remove divergent part

Solve $\Delta q = \frac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$

Elliptic equation needs Poisson solver (global computation)

Non-hydrostatic solver: algorithm

- Pressure correction method

$$p = p_a + p_H + q$$

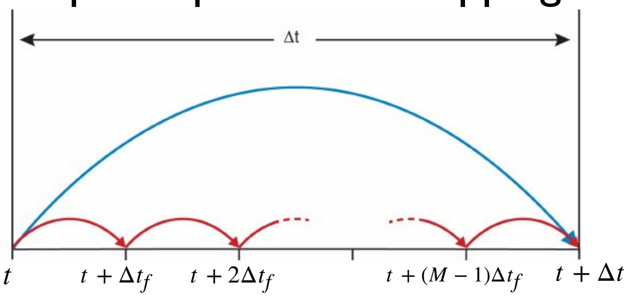
Homogeneous linearized equations

$$\begin{aligned} \partial_x u &+ \partial_z w = 0 \\ \partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\ \partial_t w &= -\partial_z q/\rho_0 \\ \\ \partial_t \eta &= w(0) = -H\partial_x \overline{u} \\ w|_{z=-H} &= 0 \\ q\,|_{z=0} &= 0 \end{aligned}$$

$$u = \bar{u} + u'$$

depth-averaged (barotropic) flow

Split-explicit time-stepping



Non-hydrostatic solver: algorithm

Pressure correction method

$$p = p_a + p_H + q$$

Homogeneous linearized equations

$$\begin{aligned}
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w|_{z=-H} &= 0 \\
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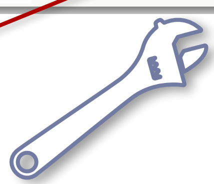
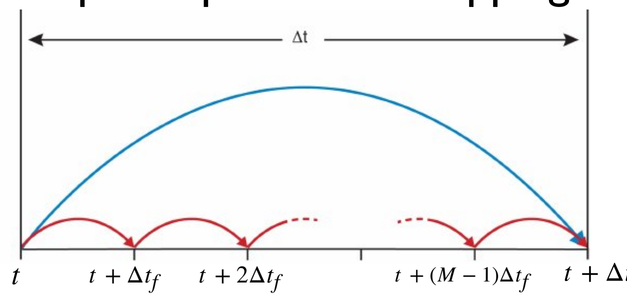
\end{aligned}$$



Split-explicit time-stepping



$$u = \bar{u} + u'$$

Compute $\bar{u}^{n+1}$ from barotropic equations

Correct velocity field to remove divergent part

$$u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q$$

However : $\overline{u}^{n+1} \neq \overline{u^{n+1}}$

Solution 1 : change boundary condition on q to $\partial_z q|_{z=0} = 0$

$$\Rightarrow \overline{u}^{n+1} = \overline{\widetilde{u}^{n+1}} = \overline{u^{n+1}}$$

Non-hydrostatic solver: algorithm

- Pressure correction method

  - 2D/3D consistency :
    - Prevents resolution of short surface waves
  - Poisson solver:
    - Complexity in sigma coordinates
    - Parallelization issues with global computations

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach (Auclair et al., 2018)

"While acoustic waves are in general entirely negligible, the effects of the approximations may not be."

Dukowics (2013)

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach

$$p = p_a + p_H + c_s^2 \delta\rho$$

Homogeneous linearized equations

$$\begin{aligned}
\partial_t u &= -g\partial_x \eta - c_s^2 \partial_x \delta\rho \\
\partial_t w &= -c_s^2 \partial_z \delta\rho \\
\partial_t \delta\rho &= -\rho_0(\partial_x u + \partial_z w) \\
\\
\partial_t \eta &= w|_{z=0} \\
w|_{z=-H} &= 0 \\
\delta\rho|_{z=0} &= 0
\end{aligned}$$

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach

$$p = p_a + p_H + c_s^2 \delta\rho$$

Split-explicit approach: the acoustic mode is integrated at the same fast step as the barotropic mode

Homogeneous linearized equations

$$\begin{aligned}
\partial_t u &= -g\partial_x \eta - c_s^2 \partial_x \delta\rho \\
\partial_t w &= -c_s^2 \partial_z \delta\rho \\
\partial_t \delta\rho &= -\rho_0(\partial_x u + \partial_z w) \\
\\
\partial_t \eta &= w|_{z=0} \\
w|_{z=-H} &= 0 \\
\delta\rho|_{z=0} &= 0
\end{aligned}$$

Semi-implicit forward-backward

$$\begin{aligned}
u^{m+1} &= u^m - \delta t \left(g\partial_x \eta^m + c_s^2 \partial_x \delta\rho^m \right) \\
w^{m+1} &= w^m - \delta t c_s^2 \partial_z \left(\delta\rho^{m+\theta} \right) \\
\delta\rho^{m+1} &= \delta\rho^m - \rho_0 \delta t \left(\partial_x u^{m+1} + \partial_z w^{m+\theta} \right) \\
\eta^{m+1} &= \eta^m + \delta t (w|_{z=0})^{m+\theta}
\end{aligned}$$

local computation

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach

Physics

- Solves short surface waves
- Solves mixed acoustic-gravity waves (tsunami precursor)

Numerics

- High-order pressure gradient → accuracy for internal waves

Performances

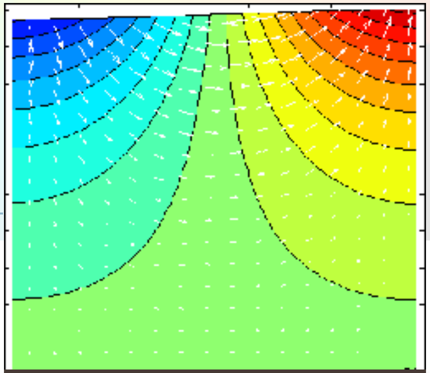
- Same fast step as hydrostatic code because of :
  - ✓ possible reduction of c_S
  - ✓ semi-implicit treatment
- Good scalability

COST: NH/H ~ 3

$$c_s \gtrsim 5\sqrt{gh}$$

Scalability

local (NBQ) / global (NH)

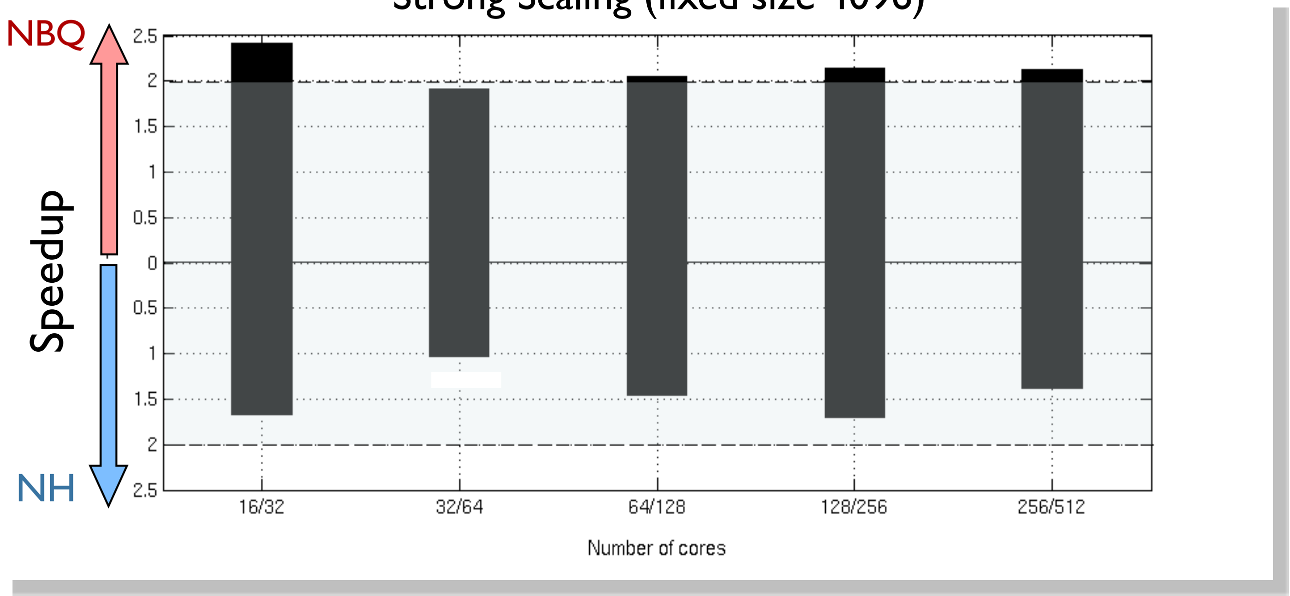


Slosh waves

$$\text{Speedup} = \frac{T(N)}{T(2N)}$$

N : nb processors

Strong Scaling (fixed size 4096)



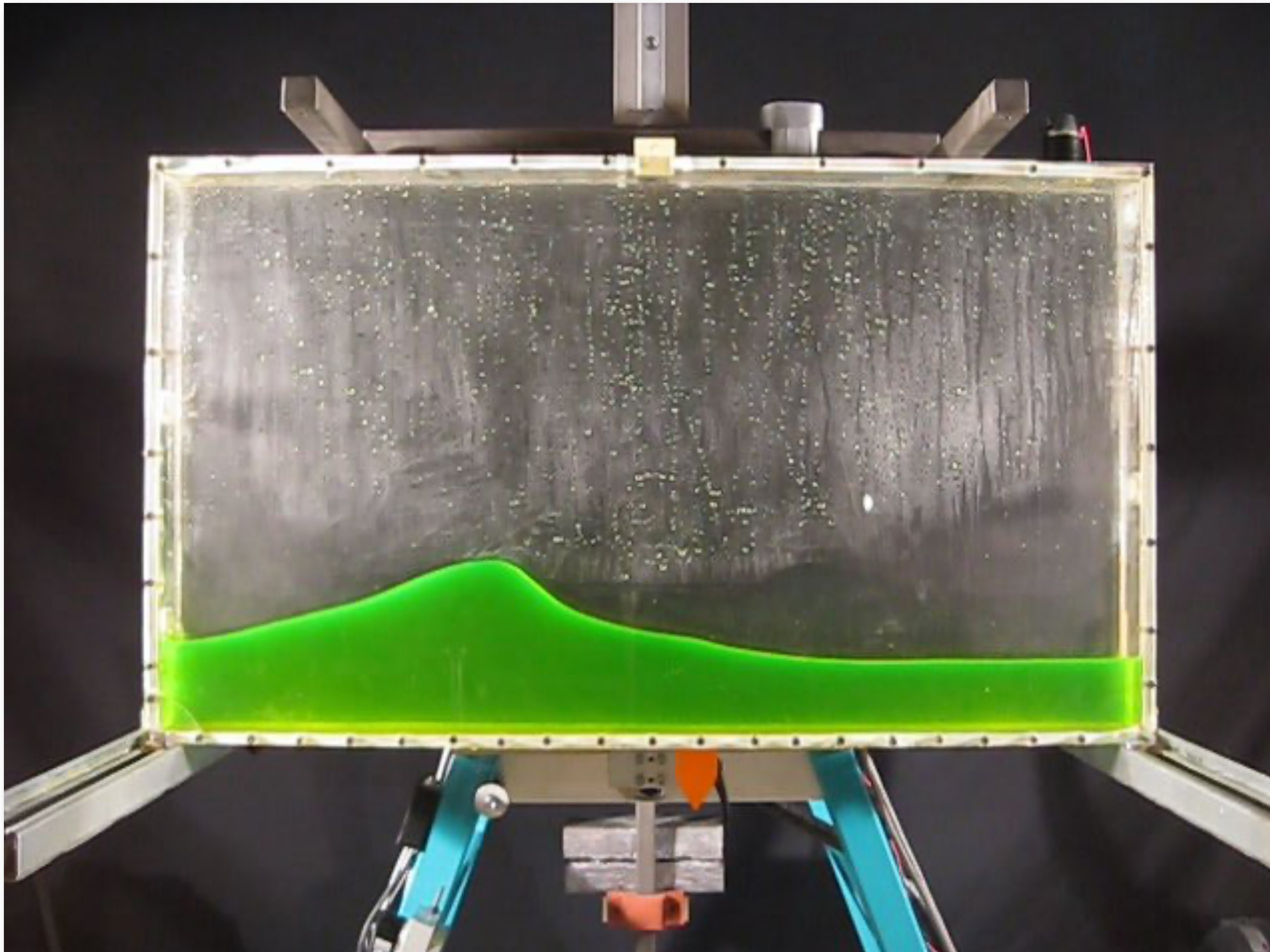
Applications

External and internal waves

Eddies, instabilities and mixing

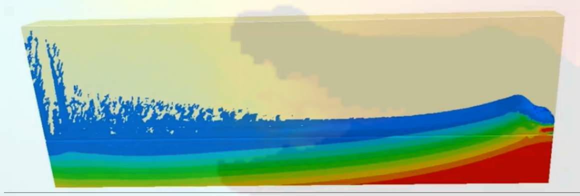
Nearshore circulation

Wave propagation experiments: CROCO test cases



CROCO test cases: TANK

Chen et al. (2003)



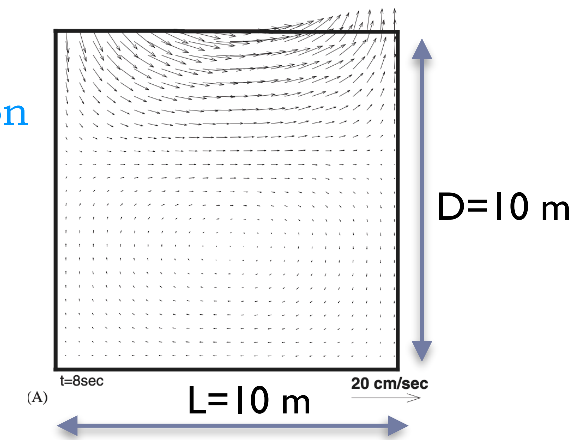
Standing wave caused by a sinusoidal free-surface set-up

20 cm resolution

$$\eta_i = a \cos kx$$

$$a = 1\ mm$$

$$k = \pi / L$$



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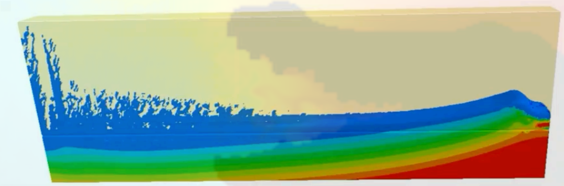

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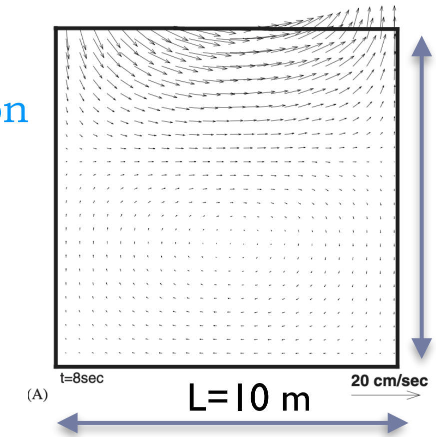
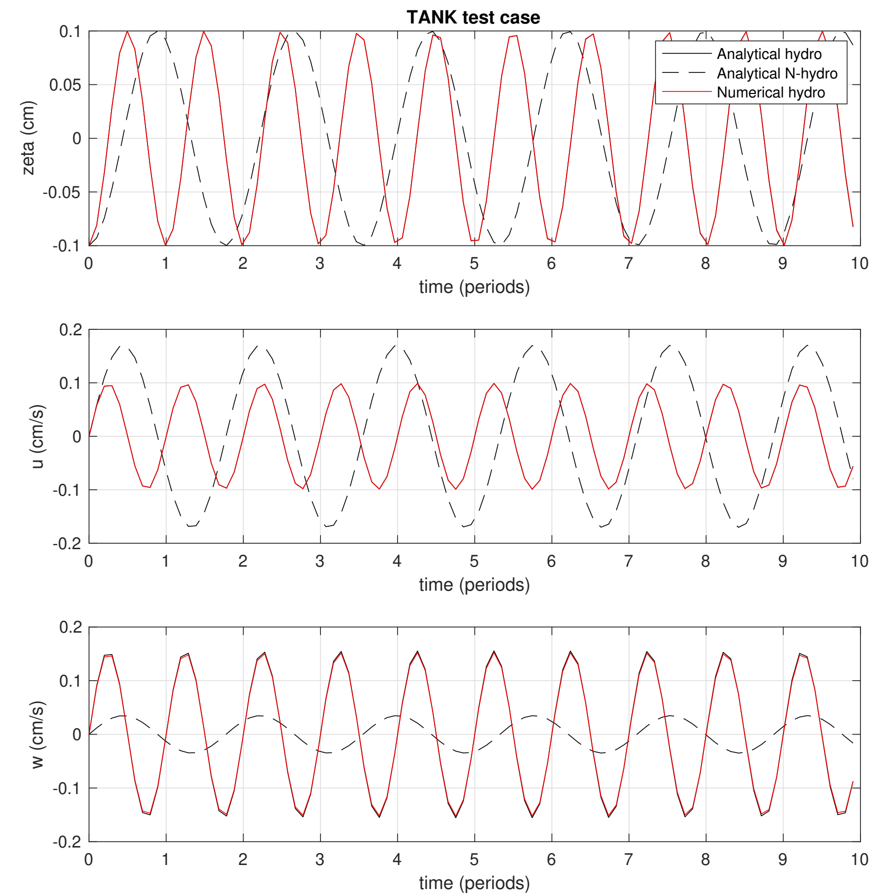


H Waves

$T \sim 2.0\ s$

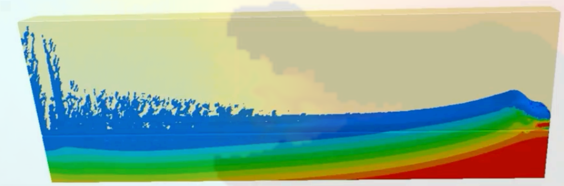
$$\begin{cases} \sigma = k\sqrt{gD} \\ \eta = a \cos kx \cos \sigma t \\ u = ag\dfrac{k}{\sigma} \sin kx \sin \sigma t \\ w = -ag\dfrac{k^2}{\sigma} \cos kx \sin \sigma t\ \ z \end{cases}$$

Hydrostatic Case



CROCO test cases: TANK

Chen et al. (2003)



Standing wave caused by a sinusoidal free-surface set-up

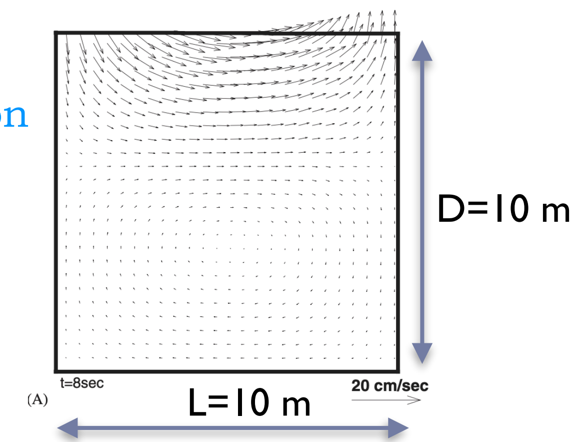
Non-Hydrostatic Case

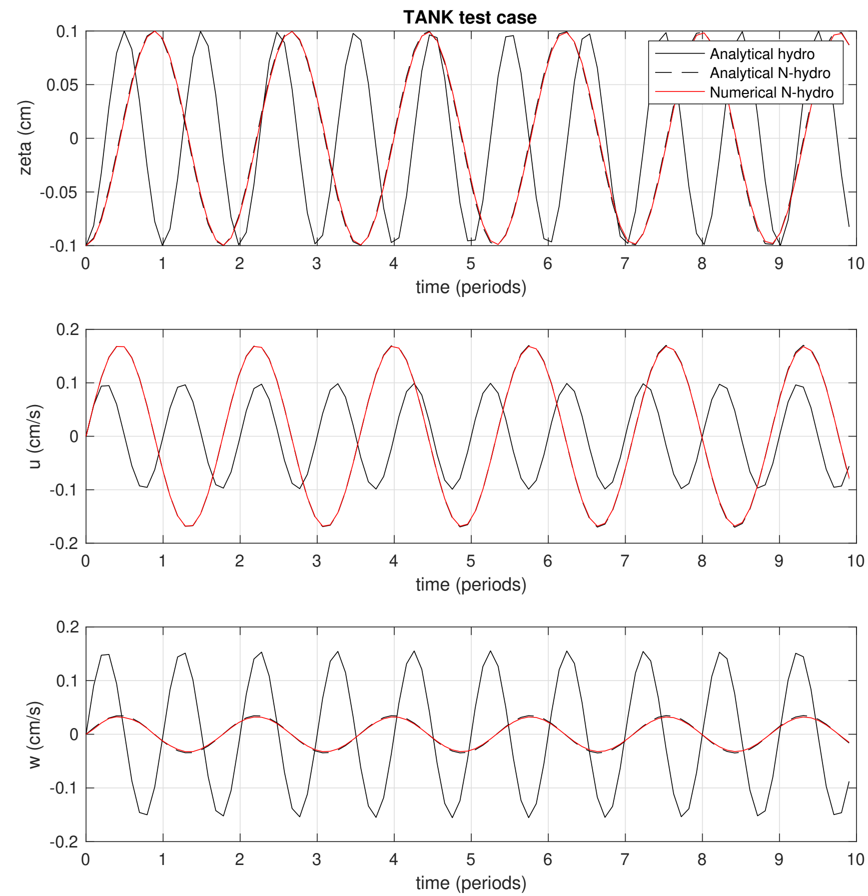
20 cm resolution

$$\eta_i = a \cos kx$$

$$a = 1\ mm$$

$$k = \pi / L$$



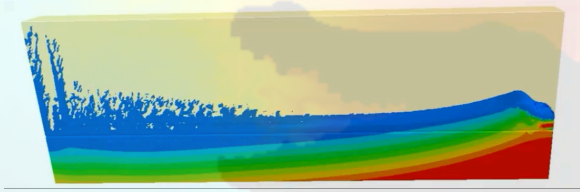


NH Waves

$$T \sim 3.6\ s$$

$$\left\{ \begin{aligned} \sigma &= \sqrt{gk \tanh kD} \\ \eta &= a \cos kx \cos \sigma t \\ u &= a\sigma \frac{\sin \sigma t}{\sinh kD} \sin kx \cosh kz \\ w &= -a\sigma \frac{\sin \sigma t}{\sinh kD} \cos kx \sinh kz \end{aligned} \right.$$

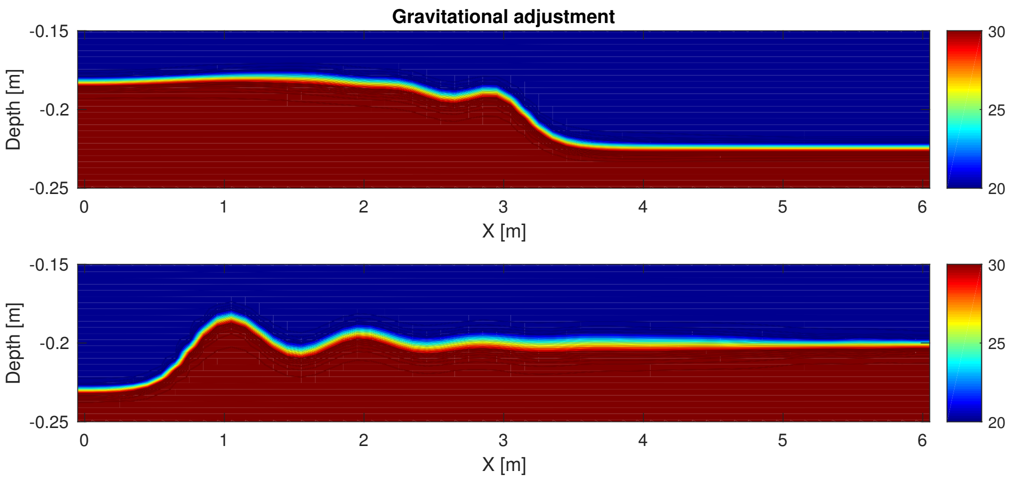
CROCO test cases: Internal Soliton

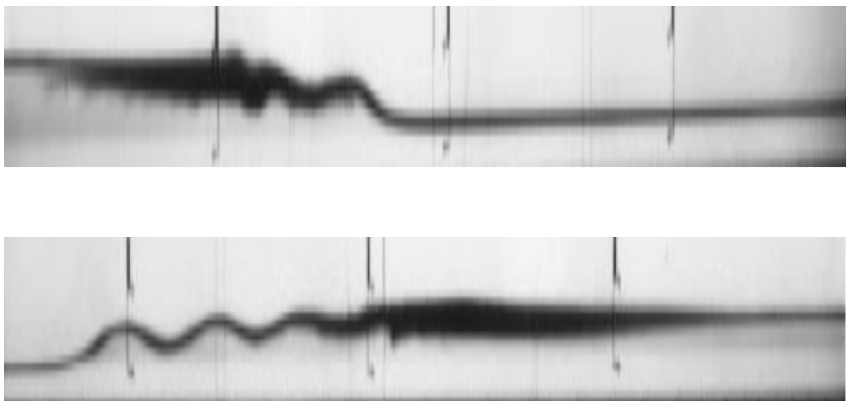


Internal Soliton from tilted interface in tank 6 m x 29 cm

CROCO 10 cm resolution

Horn et al. (2001)





Korteweg–de Vries (KdV) equation: $$\frac{\partial \eta}{\partial t} + c_0 \frac{\partial \eta}{\partial x} + \alpha \eta \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0$$

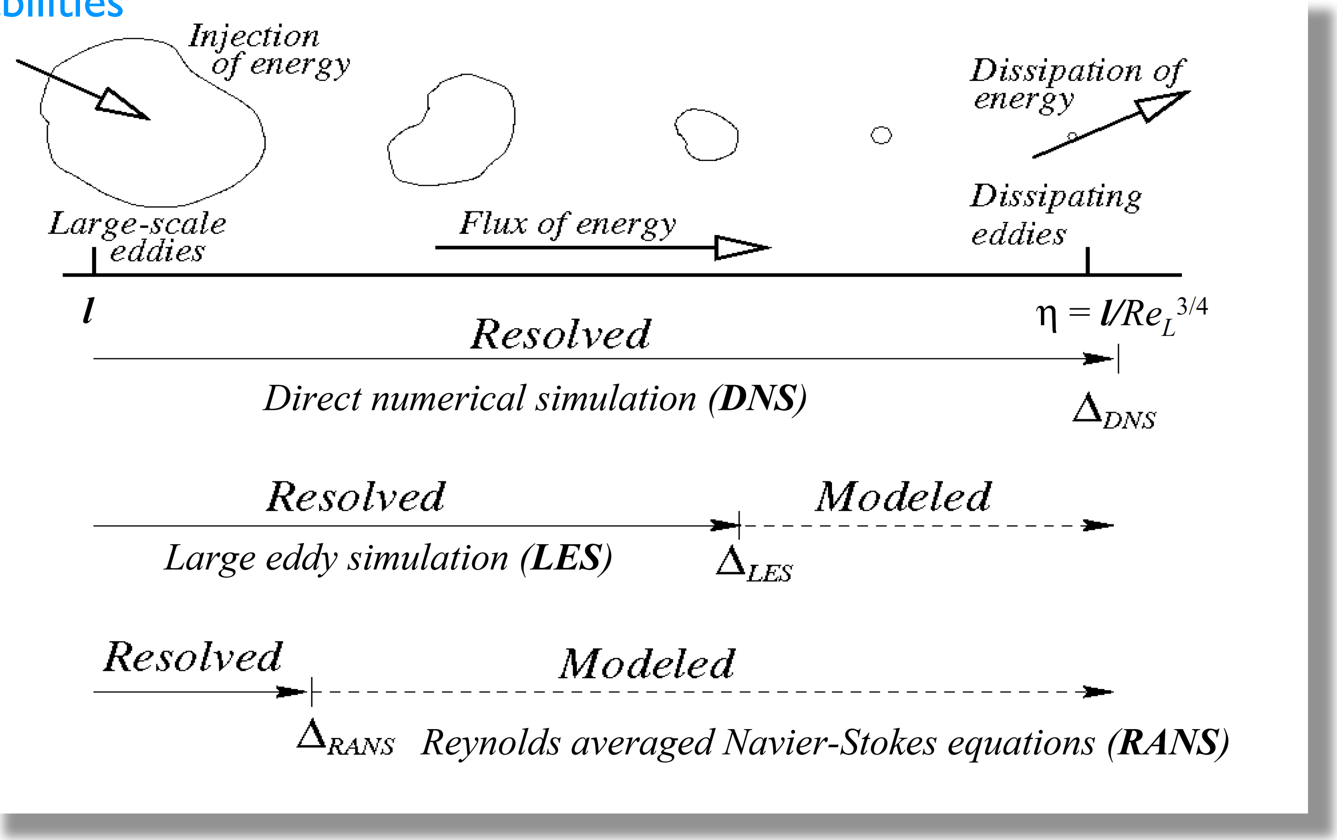
nonlinear steepening

dispersion

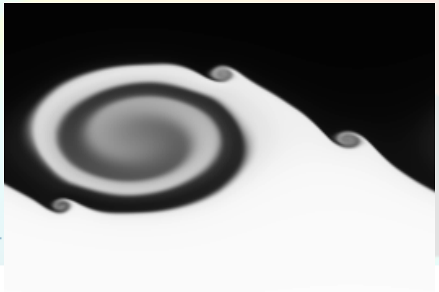
Large Eddy Simulations

Large Eddy Simulation

3D instabilities



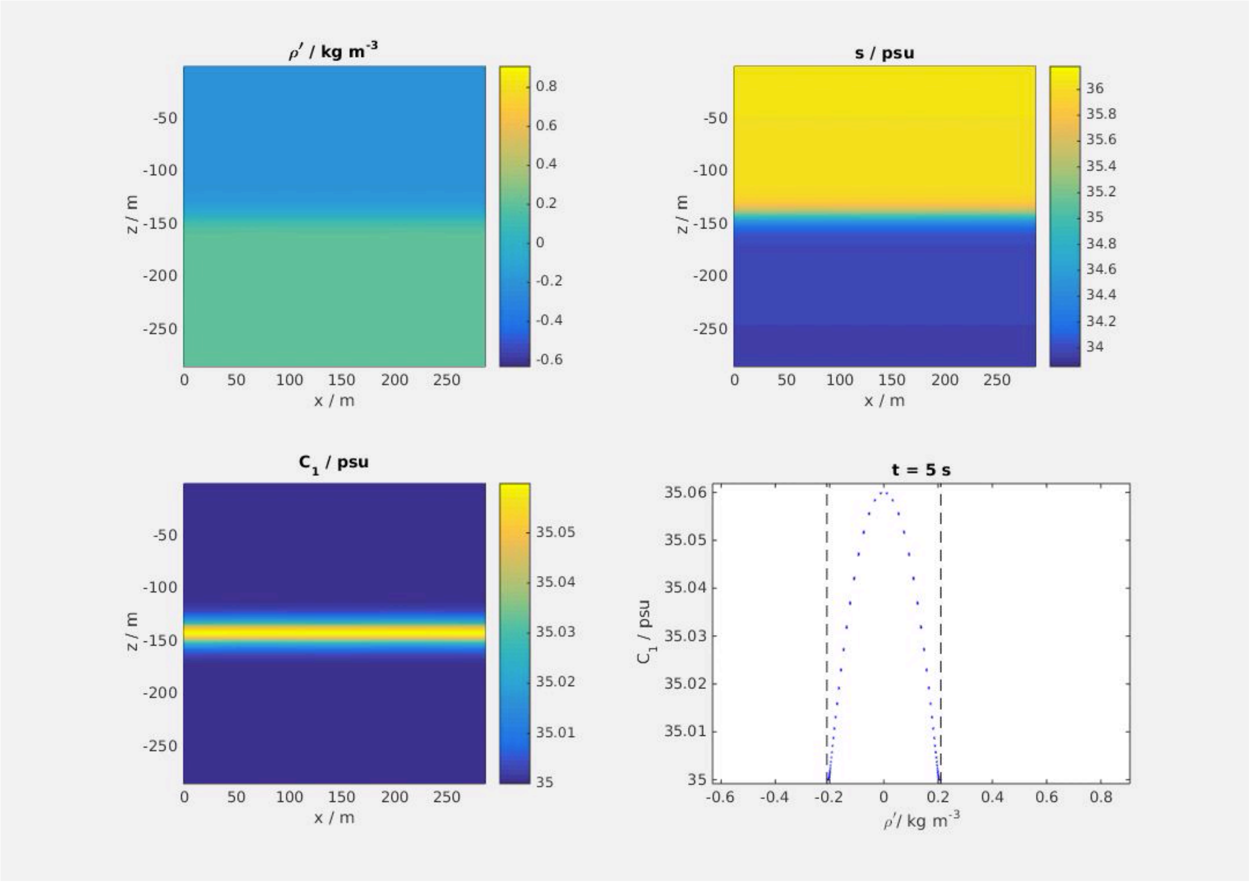
CROCO test cases: Kelvin-Helmholtz instability



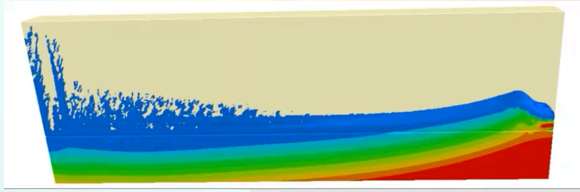
Penney et al. (2018)

Instability condition: $Ri = \frac{buoyancy}{shear} = \frac{g}{\rho}\frac{\partial\rho/\partial z}{(\partial u/\partial z)^2} < 0.25$

resolution : $1\,m$



CROCO test cases: Lock-Exchange

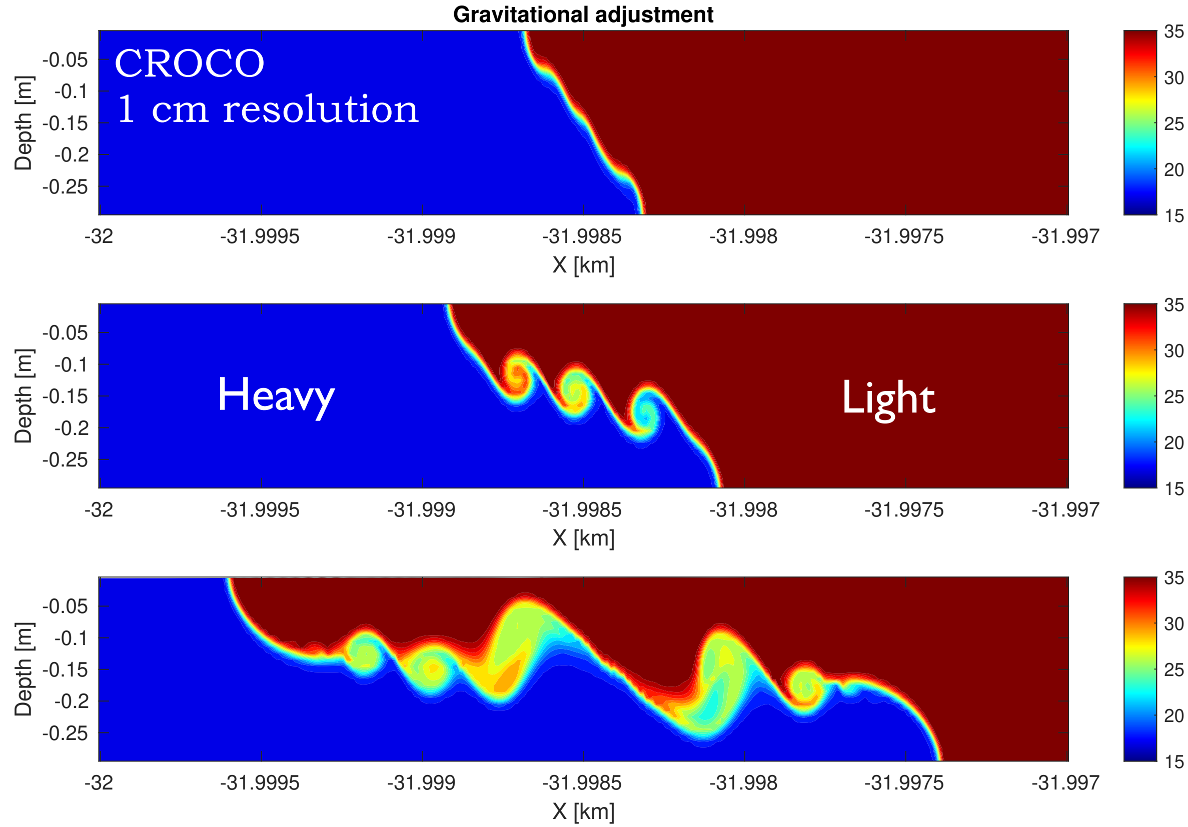


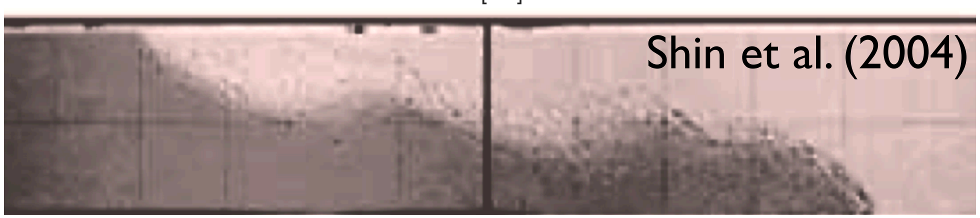
Front propagates at speed:

$$U = 0.5\sqrt{g'H}$$

$$g' = g\delta\rho/\rho_0 = 47.8\ mm^2/s$$

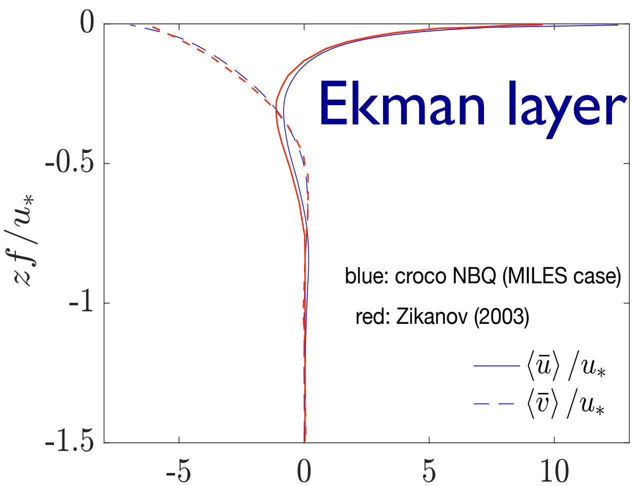
Kelvin-Helmholtz instabilities develop along the front during the gravitational adjustment

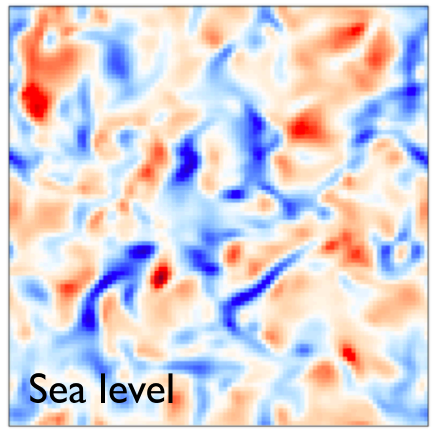


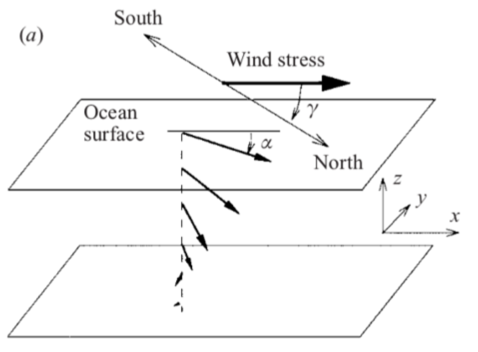


Other 3D instabilities in CROCO

N. Suzuki & B. Fox-Kemper [ICR, Ger.]; G. Xu (Washington U.) ...

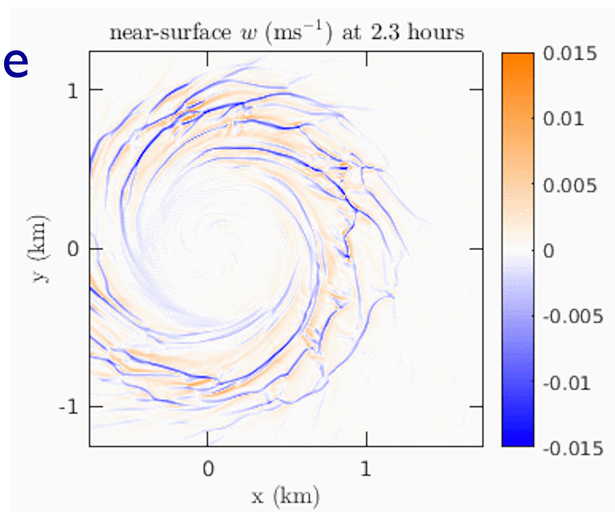




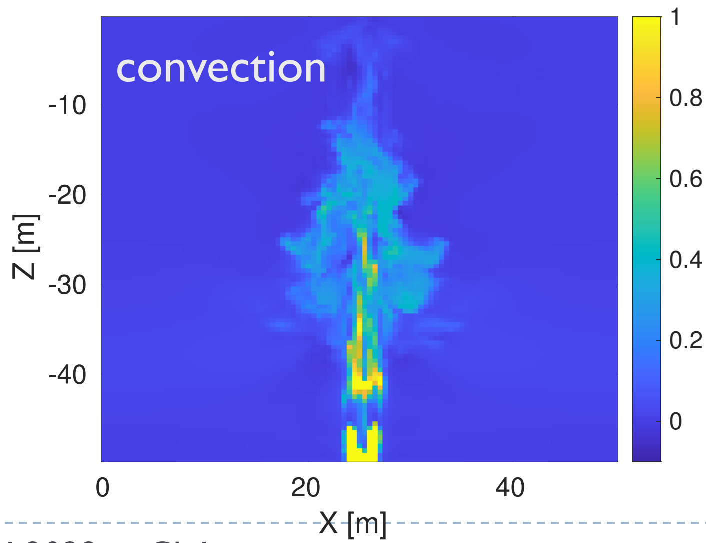


Submesoscale vortex

inertial, symmetric instabilities



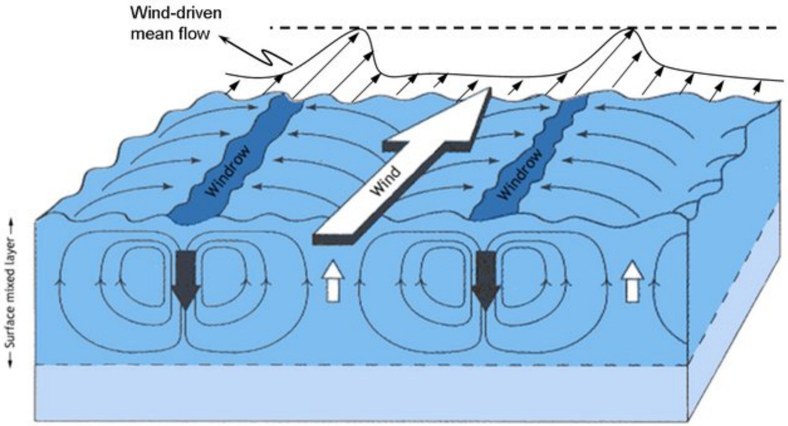
Hydrothermal plume

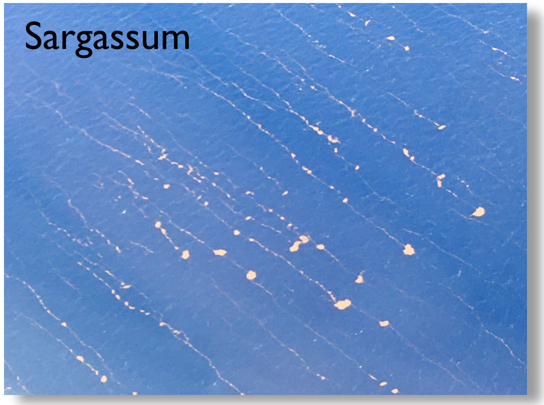


Wave effect on currents:
Langmuir turbulence

Herman et al. (2020)

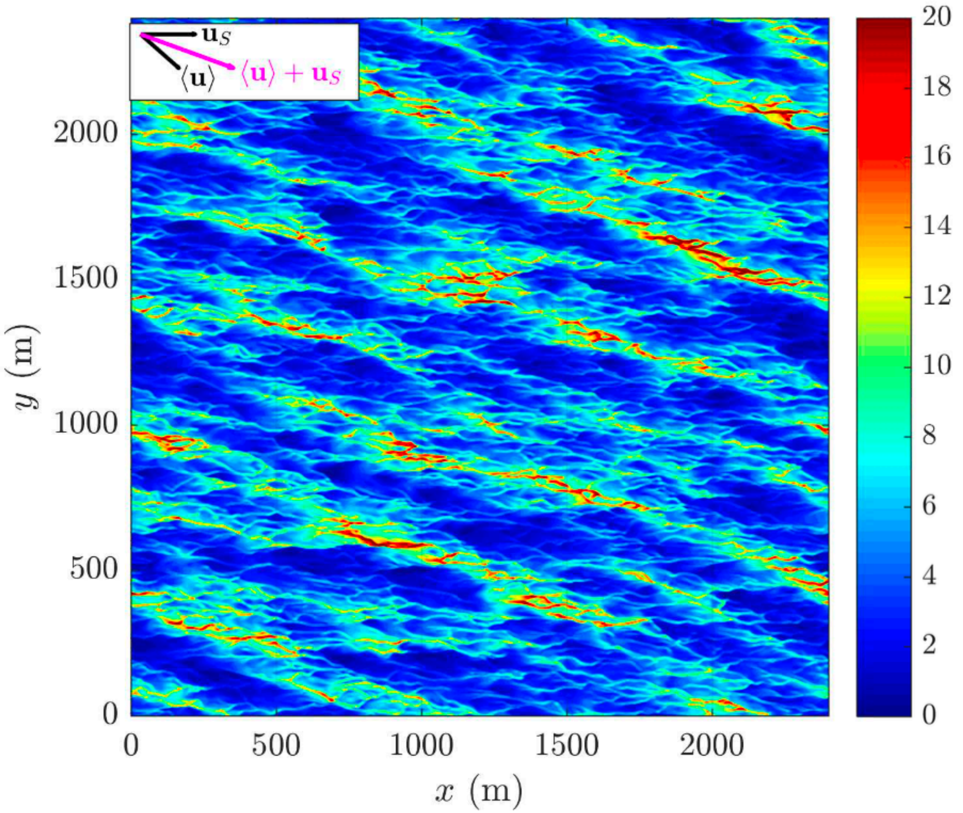
Vortex Force: $\rho\, u_S \times \xi$

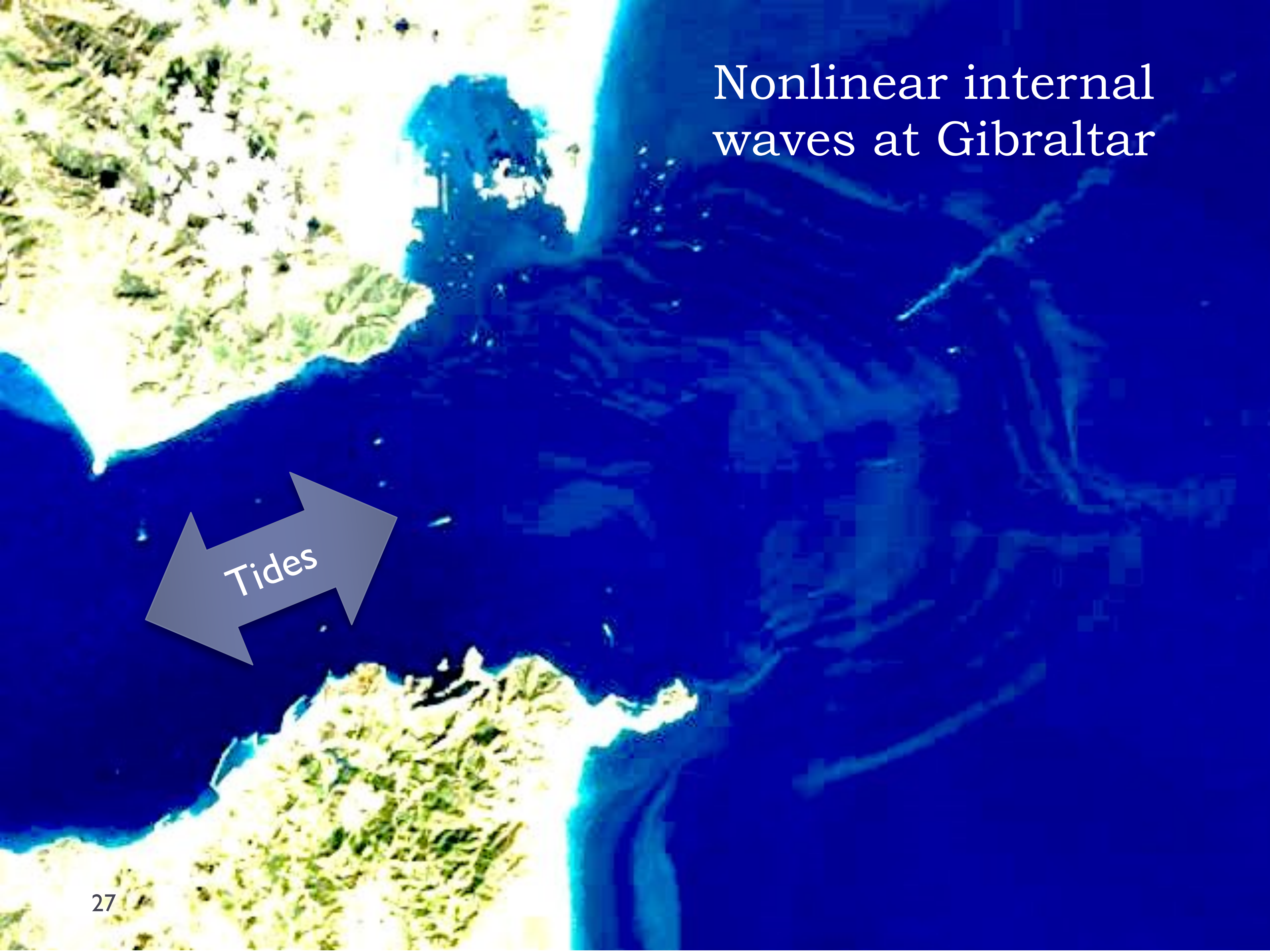




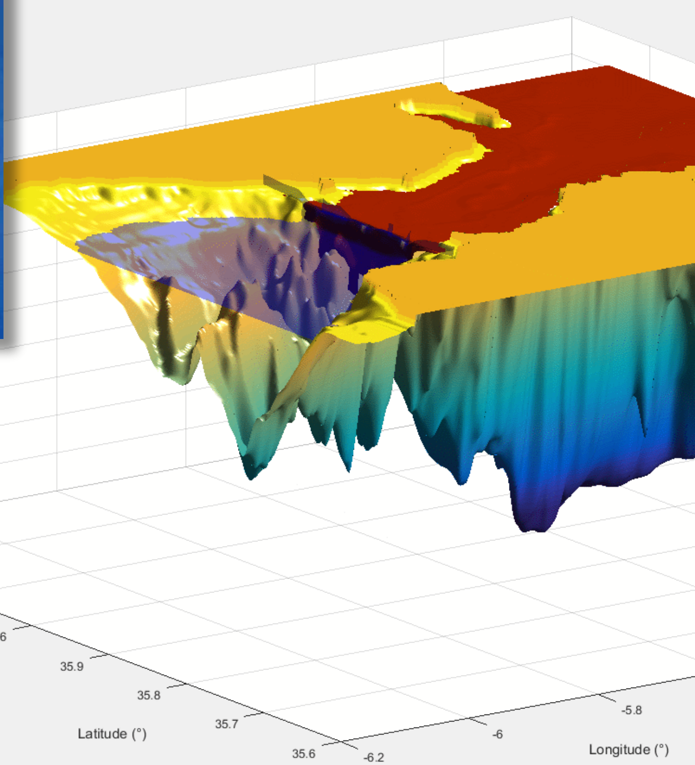
Frazil ice: LES simulation with wave-averaged equations

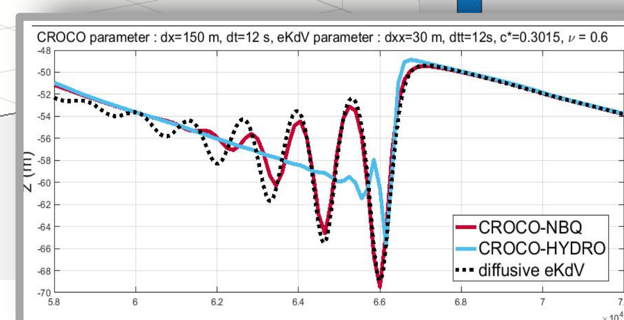
resolution : $3\,m$




Nonlinear internal
waves at Gibraltar
Tides

Nonlinear internal waves

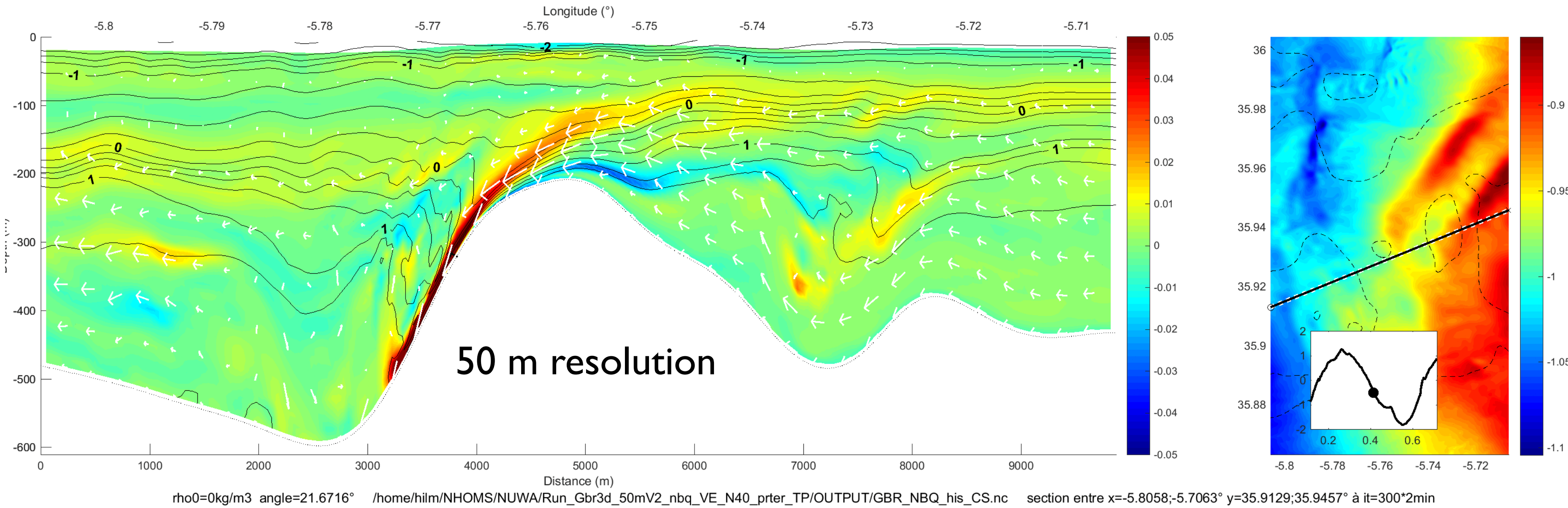




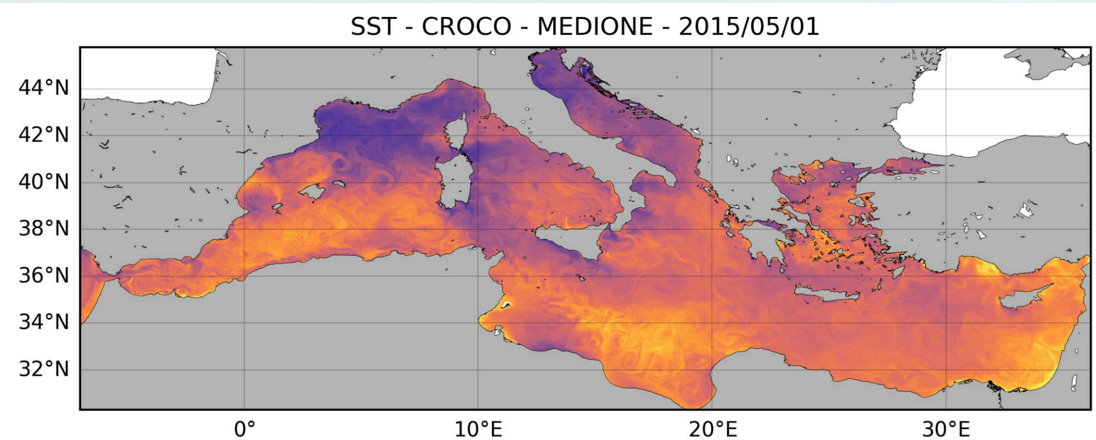
Bordois et al., 2018

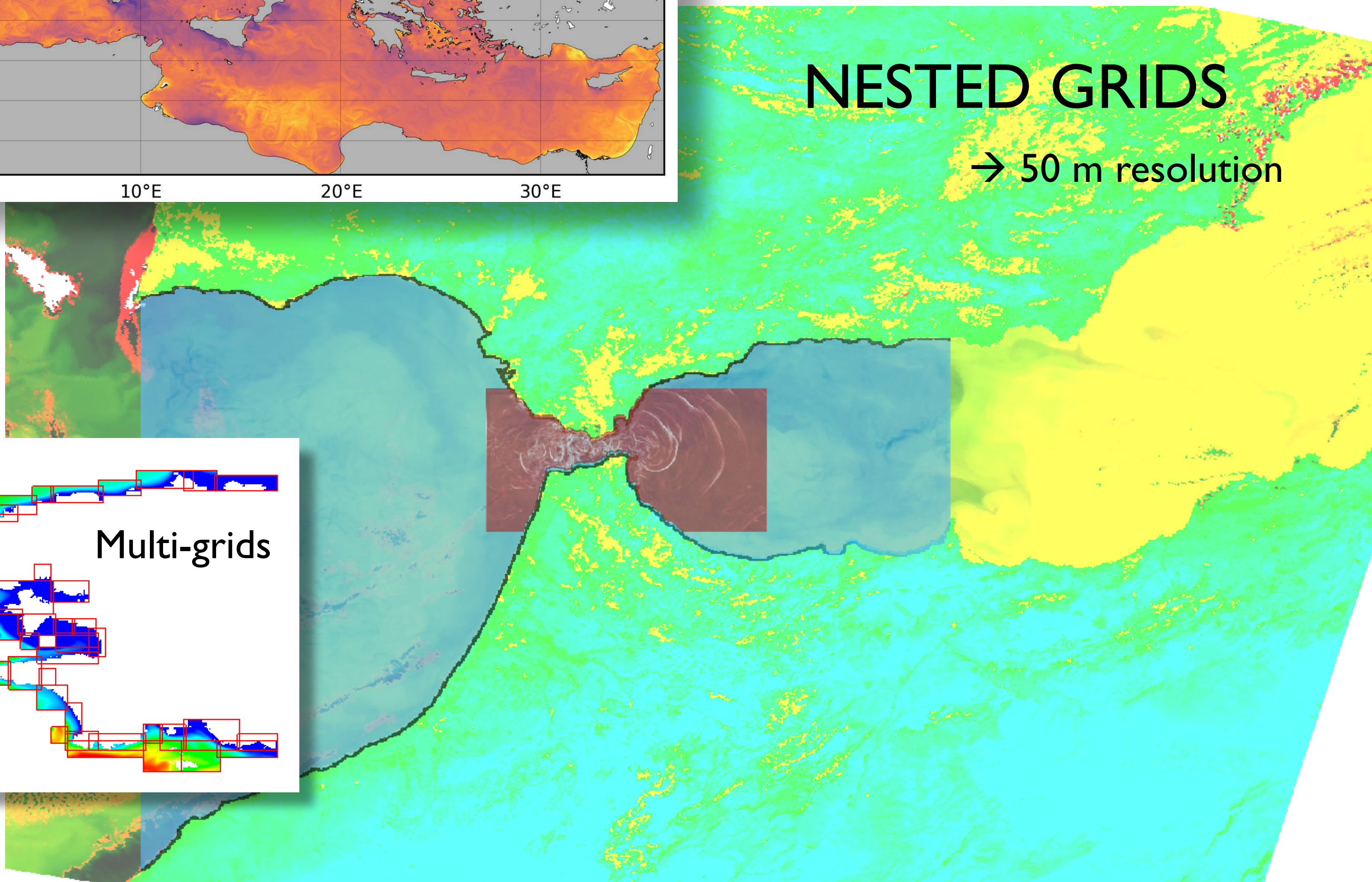
Internal hydraulic jump

Hilt et al. (2019)



Multiscale modeling





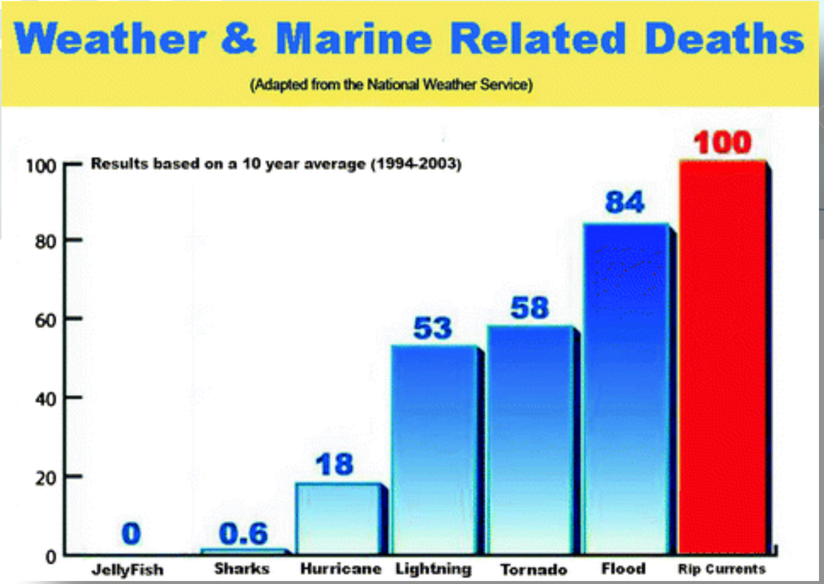
NESTED GRIDS

→ 50 m resolution

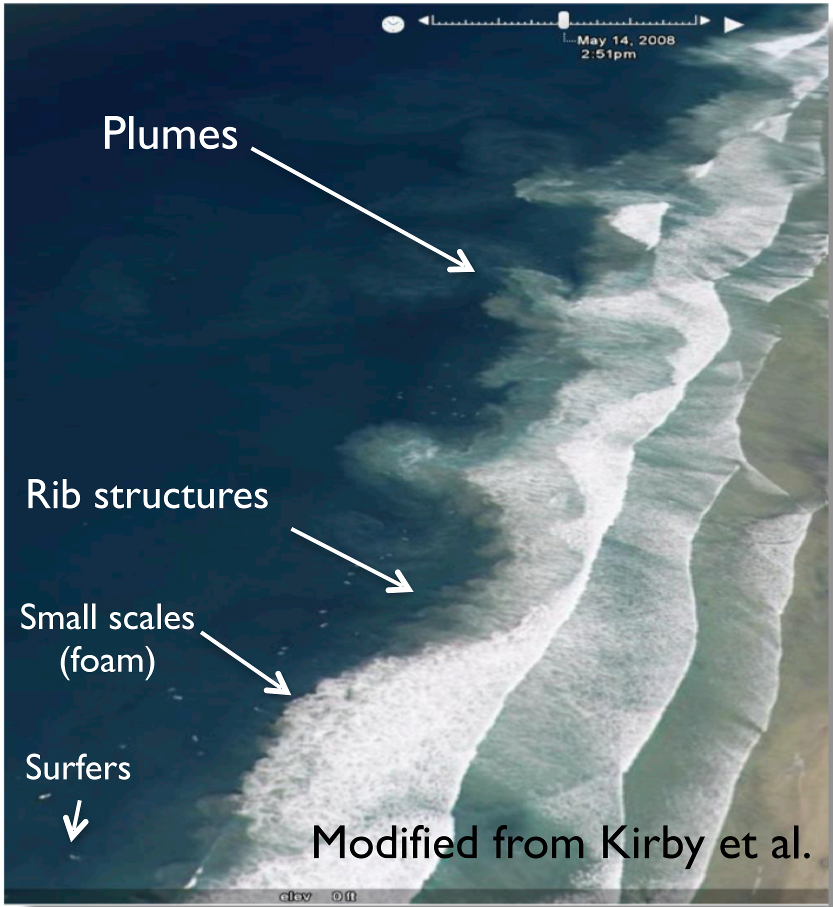
Multi-grids

Surface gravity waves & nearshore dynamics

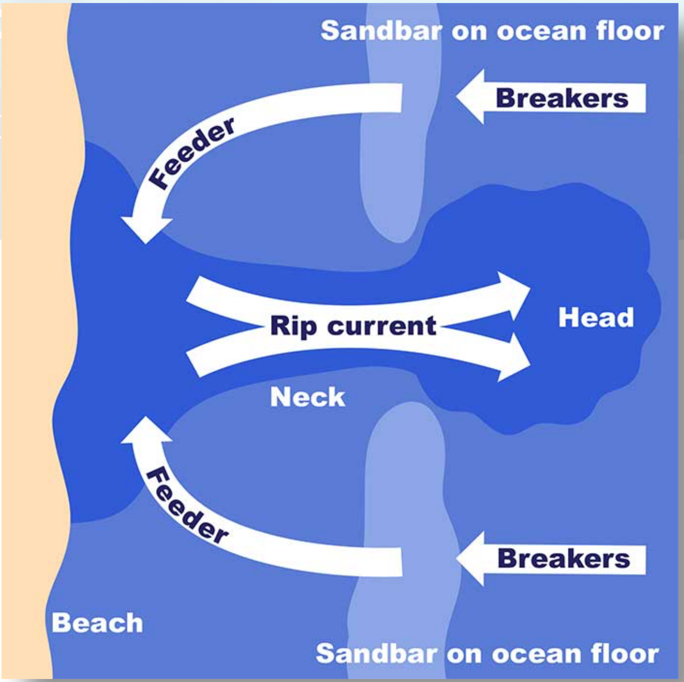
Rips and surfzone eddies



- Structure:
  - plumes, ribs, patches
- Dynamics :
  - intrinsic or forced variability?
  - 2D or 3D?
- Impacts:
  - surf hazard
  - surf mixing
  - surf-shelf exchange



3D wave-averaged modeling
Channeled rip currents



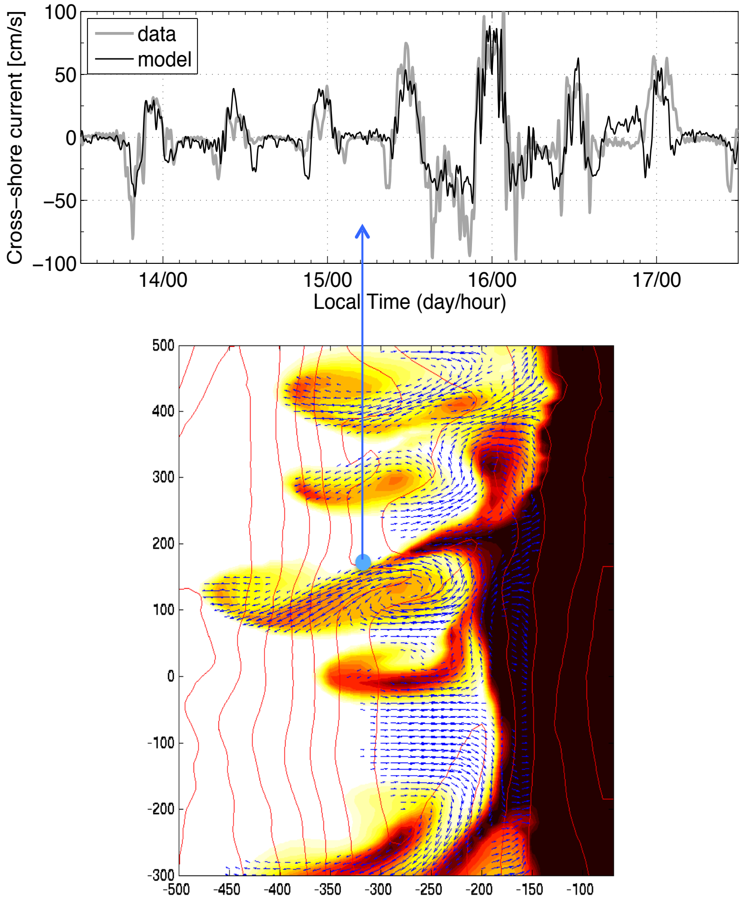
McWilliams et al. (2004)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla_\perp)\mathbf{u} + w\frac{\partial \mathbf{u}}{\partial z} + f\hat{\mathbf{z}}\times\mathbf{u} + \nabla_\perp\phi - \mathbf{F} = -\nabla_\perp\mathscr{K} + \mathbf{J} + \mathbf{F}^w,$$

$$\frac{\partial \phi}{\partial z} + \frac{g\rho}{\rho_0} = -\frac{\partial \mathscr{K}}{\partial z} + K,$$

$$\nabla_\perp\cdot\mathbf{u} + \frac{\partial w}{\partial z} = 0,$$

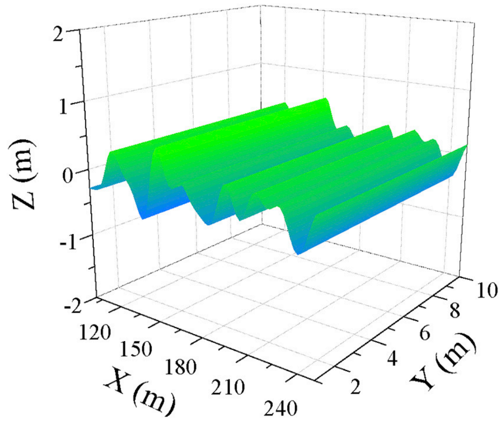
$$\frac{\partial c}{\partial t} + (\mathbf{u}\cdot\nabla_\perp)c + w\frac{\partial c}{\partial z} - \mathscr{C} = -(\mathbf{u}^{St}\cdot\nabla_\perp)c - w^{St}\frac{\partial c}{\partial z} + \frac{1}{2}\frac{\partial}{\partial z}\left[\mathscr{E}\frac{\partial c}{\partial z}\right].$$



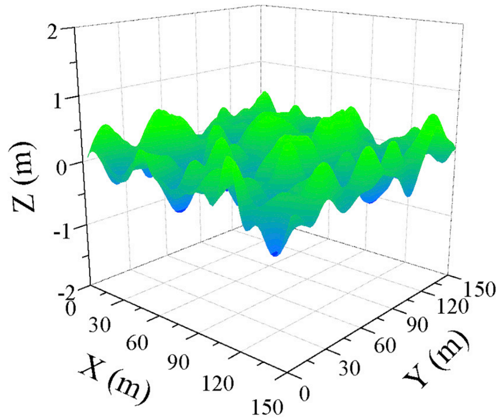
Marchesiello et al. (2015)

Wave-resolving models

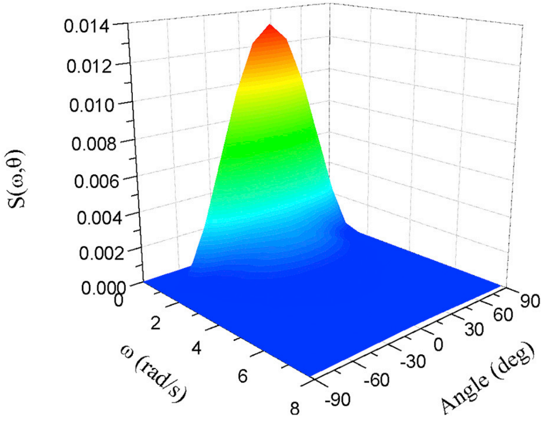
Short-crested waves



Long-crested waves



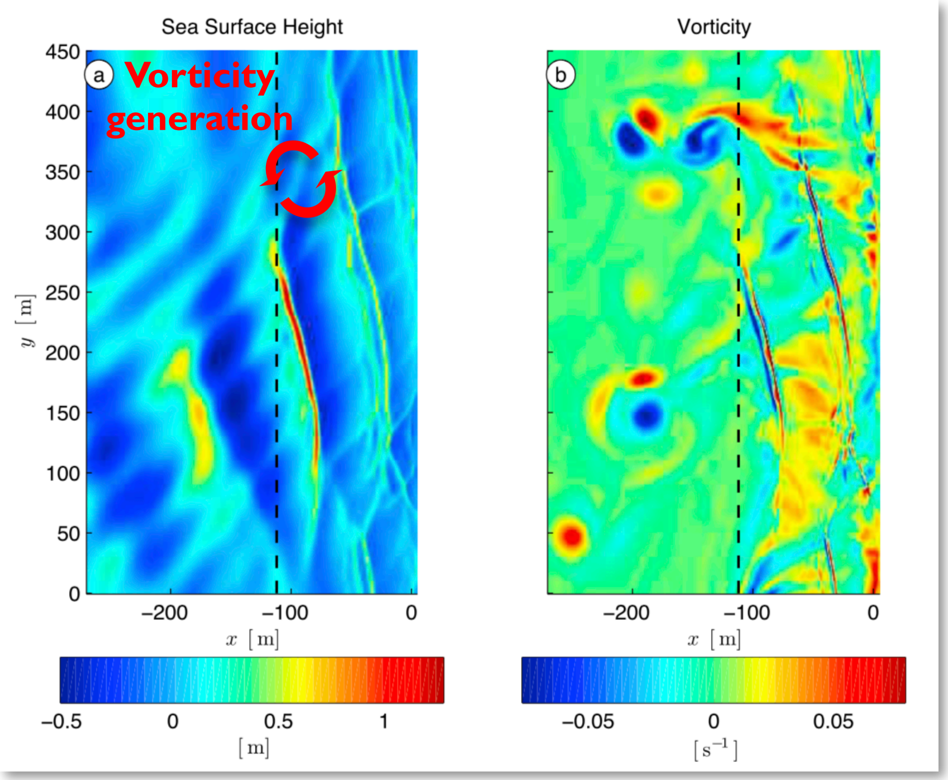
Short-crested waves



Frequency and directional spectrum

2D Wave-resolving models
Short-crested waves and flash rips

Flash rip generation by short-crested waves (Peregrine, 1998)

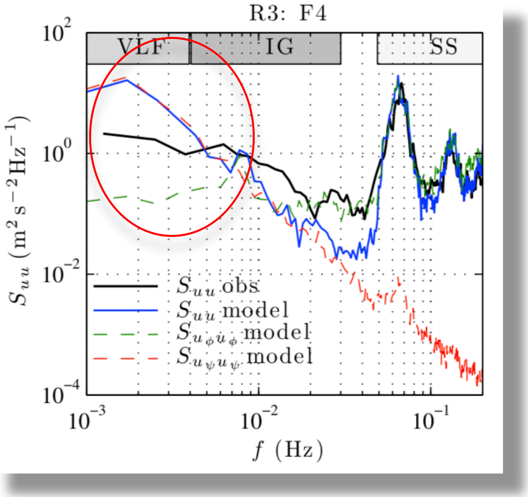


2D wave-resolving Boussinesq model
(Feddersen et al., 2011)

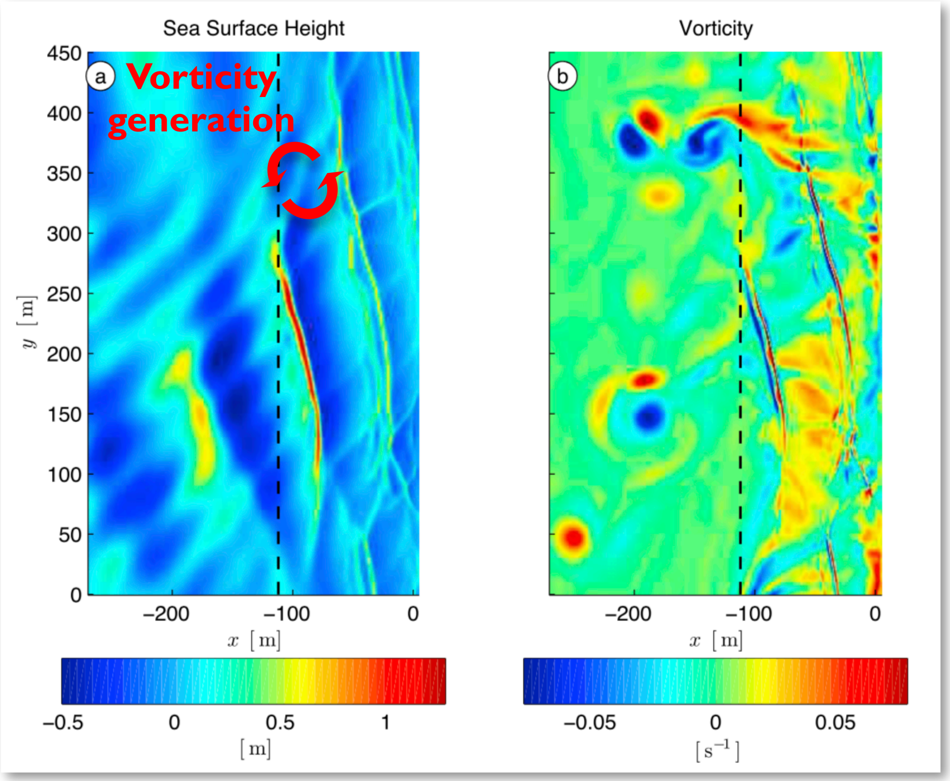
2D Wave-resolving models

Short-crested waves and flash rips

Too much energy at VLF ?



Feddersen et al. (2011)

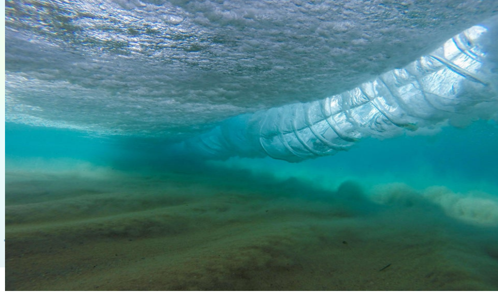


2D wave-resolving Boussinesq model
(Feddersen et al., 2011)

3D wave-resolving models

Solving or not the breaking turbulence

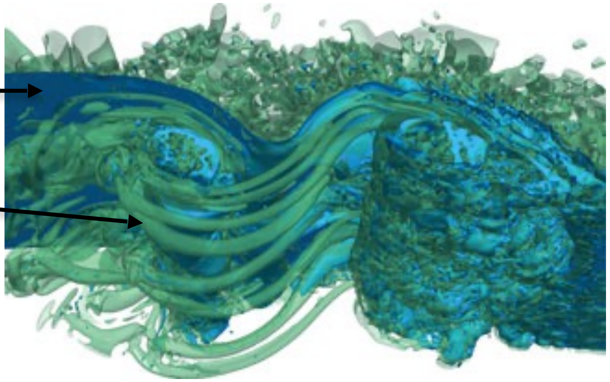
CROCO
Coastal and Regional Ocean COmmunity model



- **VOF (LES) models**: solves breaking turbulence

Roller vortices

Rib vortices



Time scale < wave period

Lubin & Glockner (2015)

- **Free-surface (RANS) models**: solves current instabilities

CROCO
NHWAVE
SWASH

Time scale > wave period

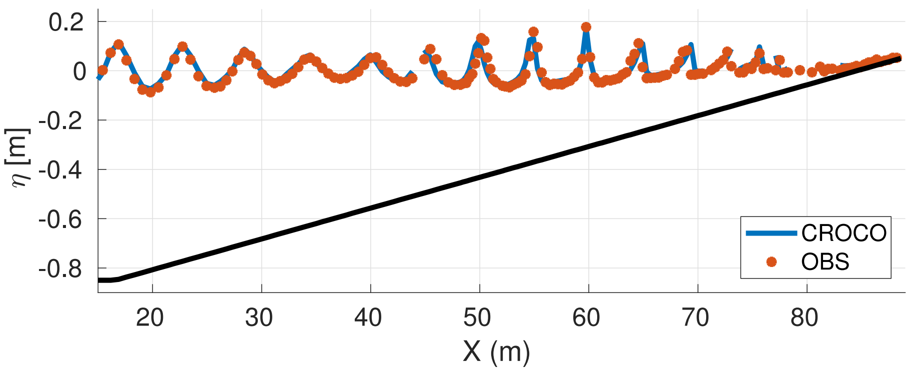
Li & Darlymple (1998)

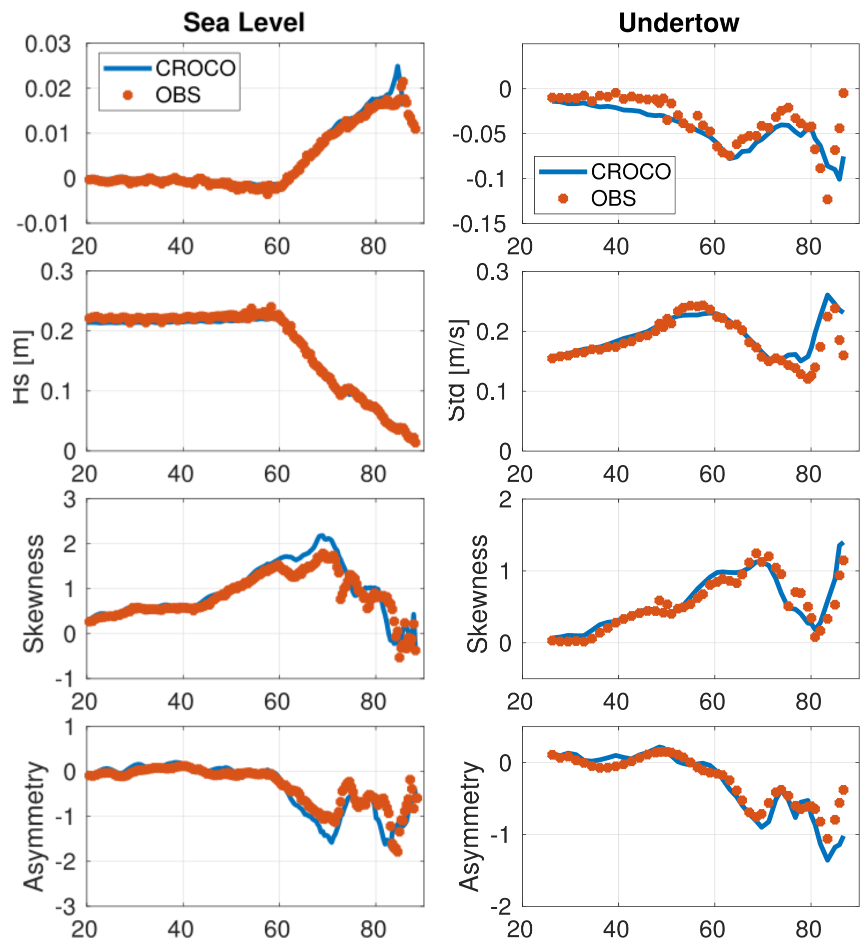
Validation with flume experiments

GLOBEX (B2) - Michalet et al. (2014)

Scheldt Wave Flume (Deltares)

- ✓ Resolution: 12 cm, 10 sigma levels
- ✓ Breaking-induced turbulence: WENO5 + *k*-ω





Large-scale flume

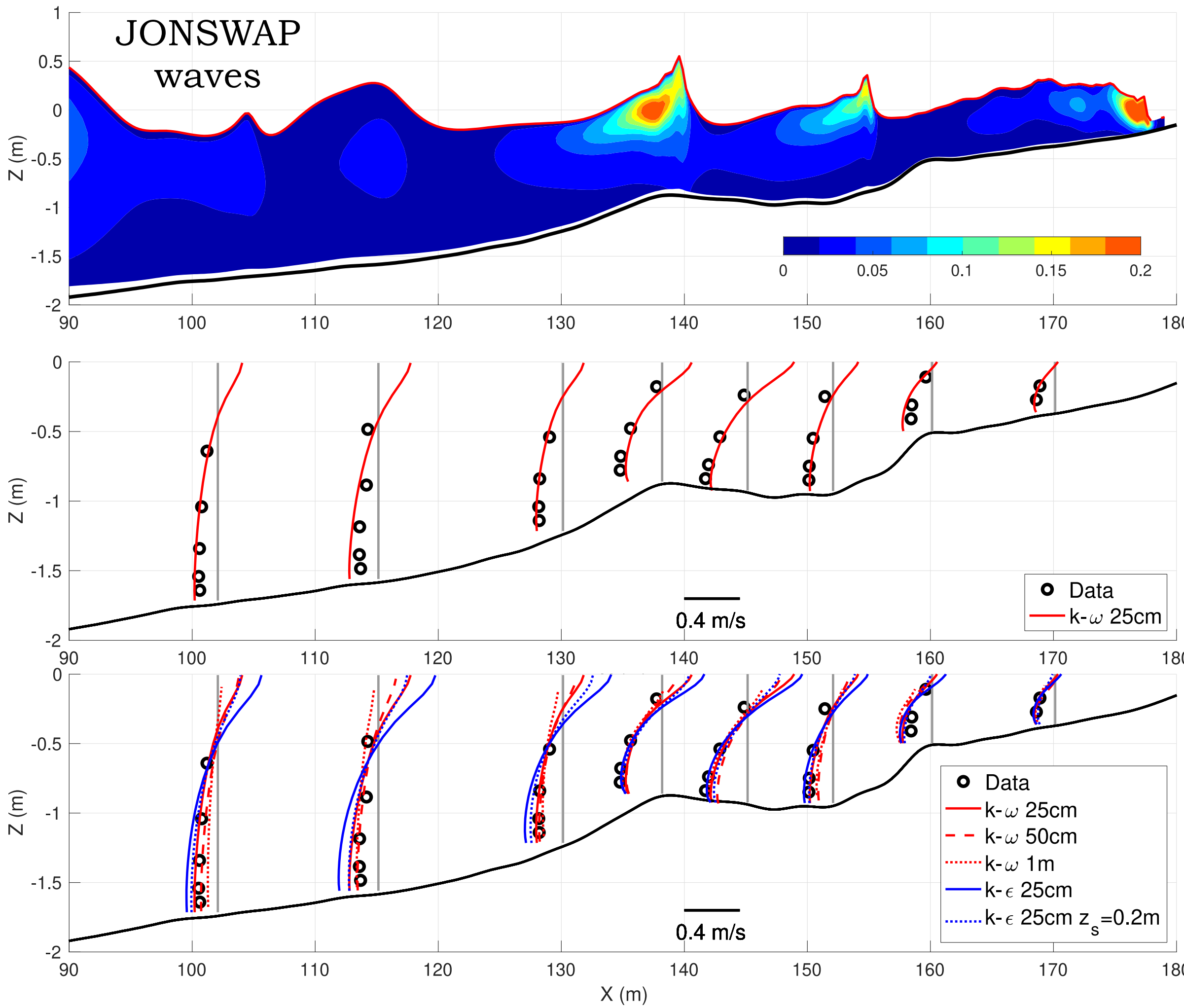
LIP-11D (1B) - Roelvink & Reniers (1995)

Delta Flume (Deltares)

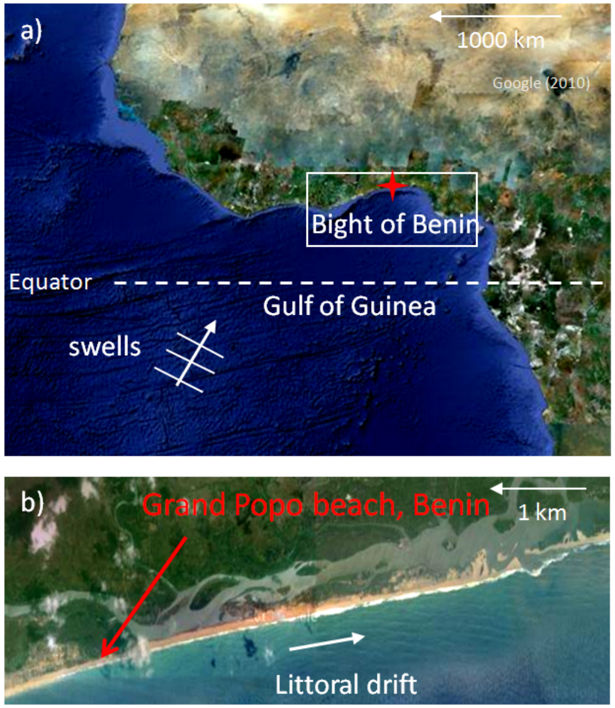
TKE

Mean U

Test of resolution and turbulence closure



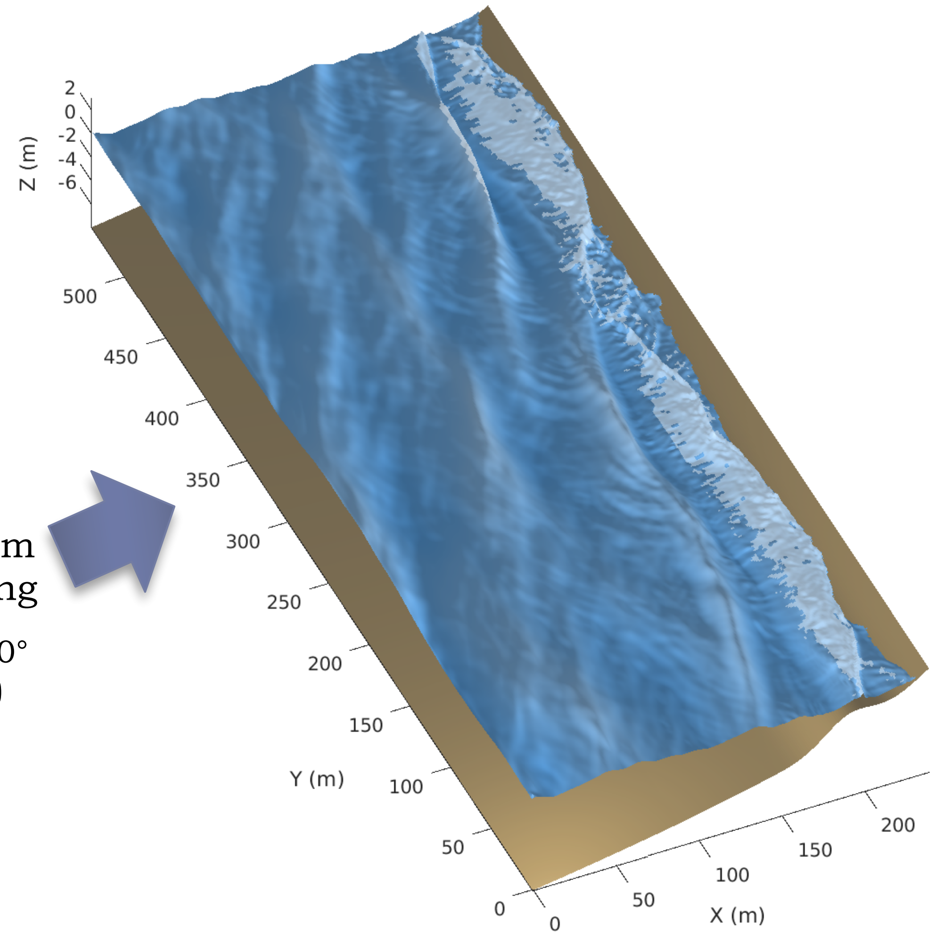
Application to a longshore-uniform beach in Grand Popo, Benin



Resolution: 50 cm, 10 lev.
SGS model: WENO5 + *k*-ω

JONSWAP wave spectrum with directional spreading

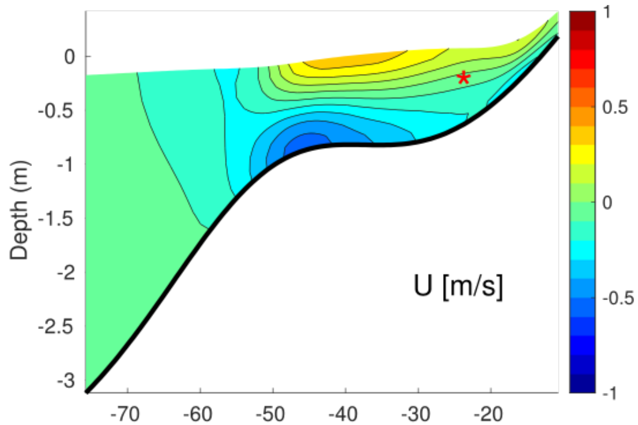
Hs=1.15 m, Tp=11 s, Dir=10°
(mid-tide, March 13 2014)



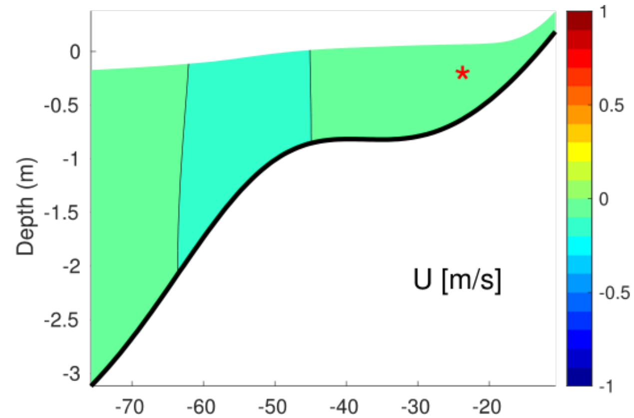
Shallow vs. Deep breaking experiments

Cross-shore currents

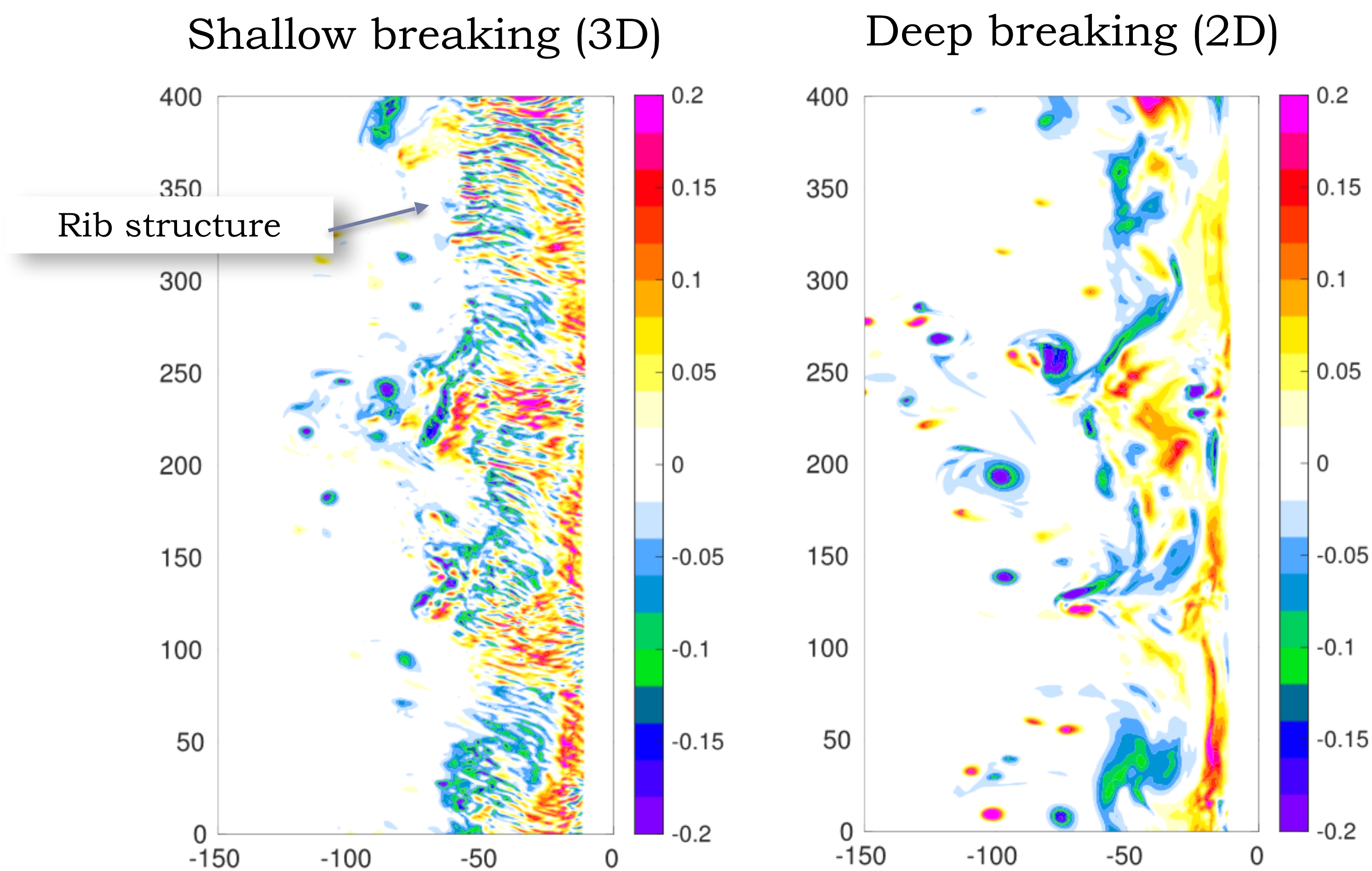
Shallow breaking (3D)



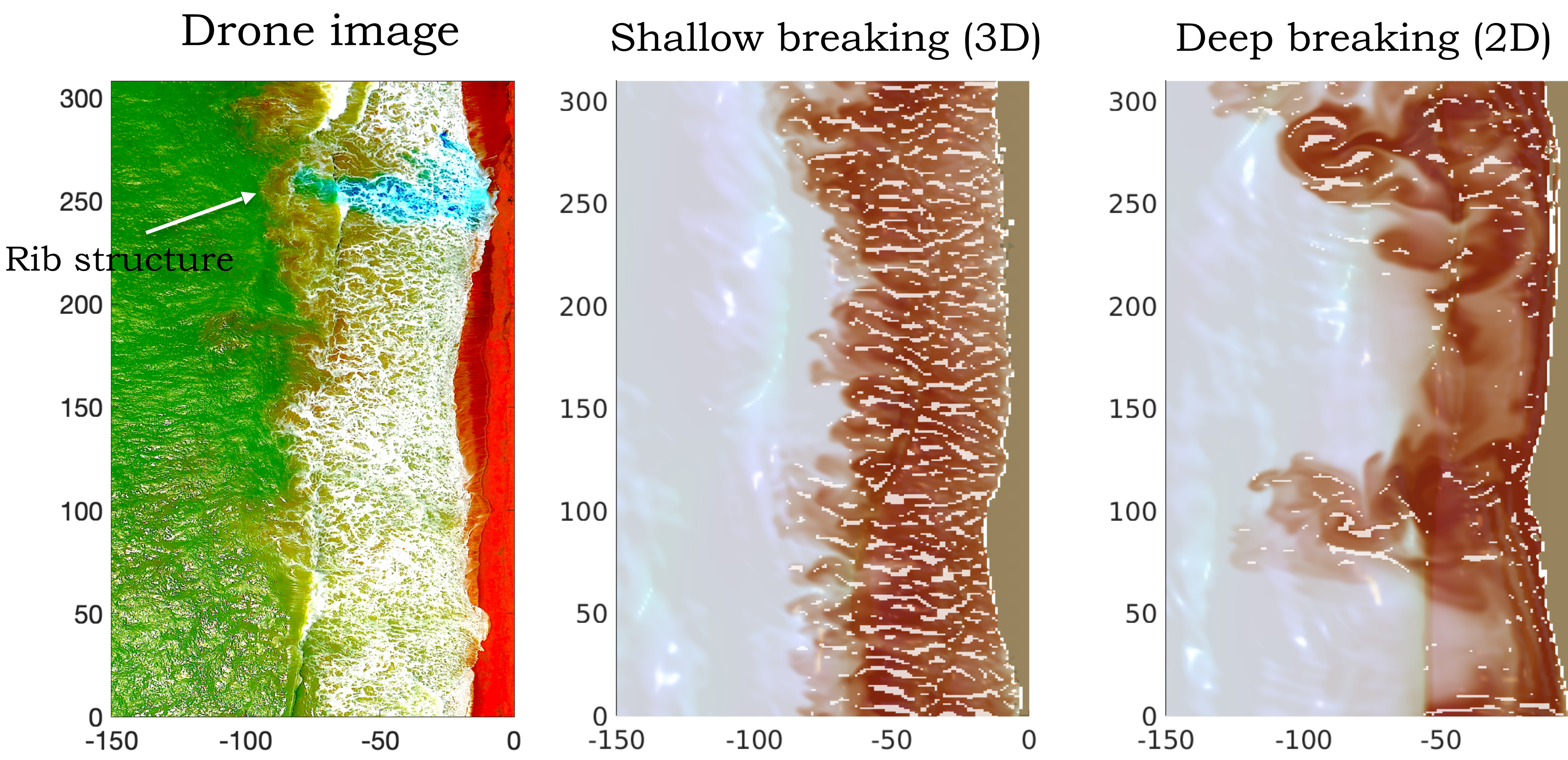
Deep breaking (pseudo-2D)



Wave-mean vertical vorticity patterns
Flash rips and mini-rips

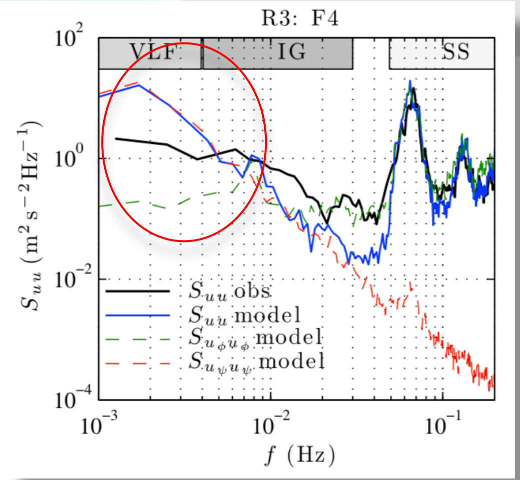


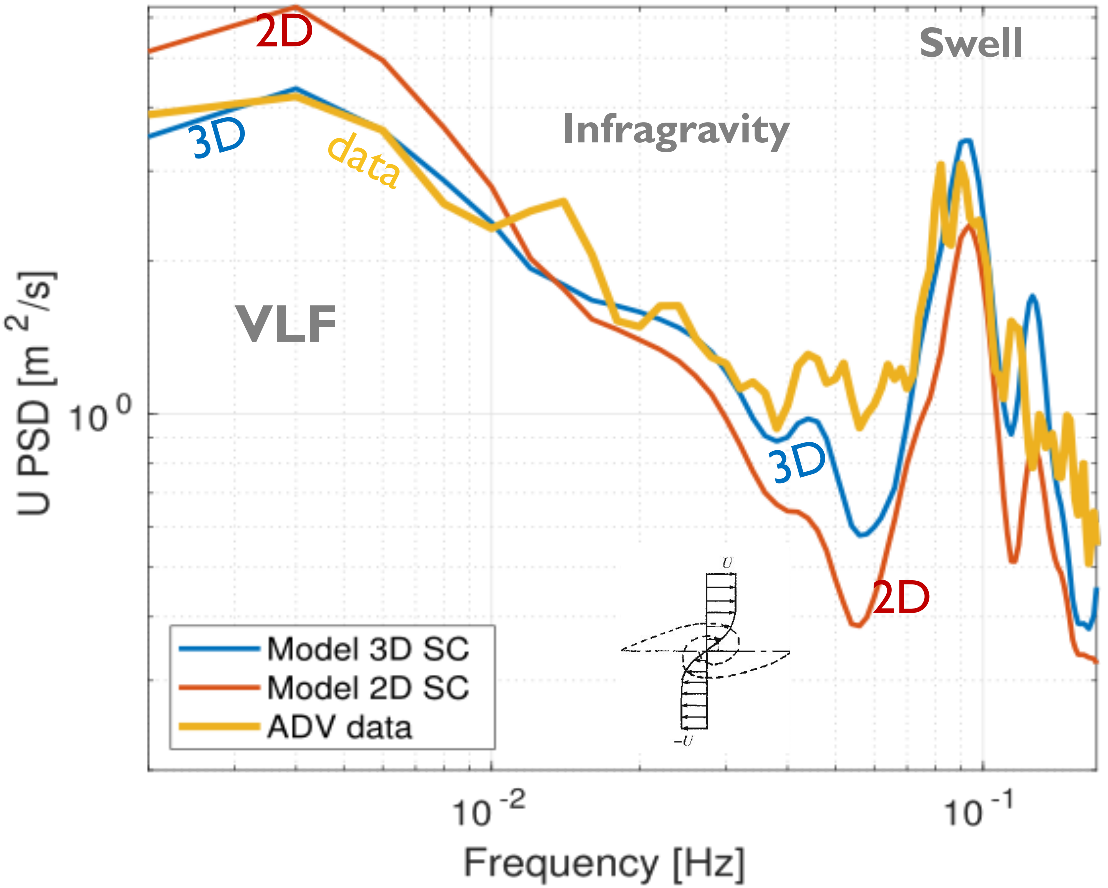
Rib structures in turbidity
with a suspended sediment model



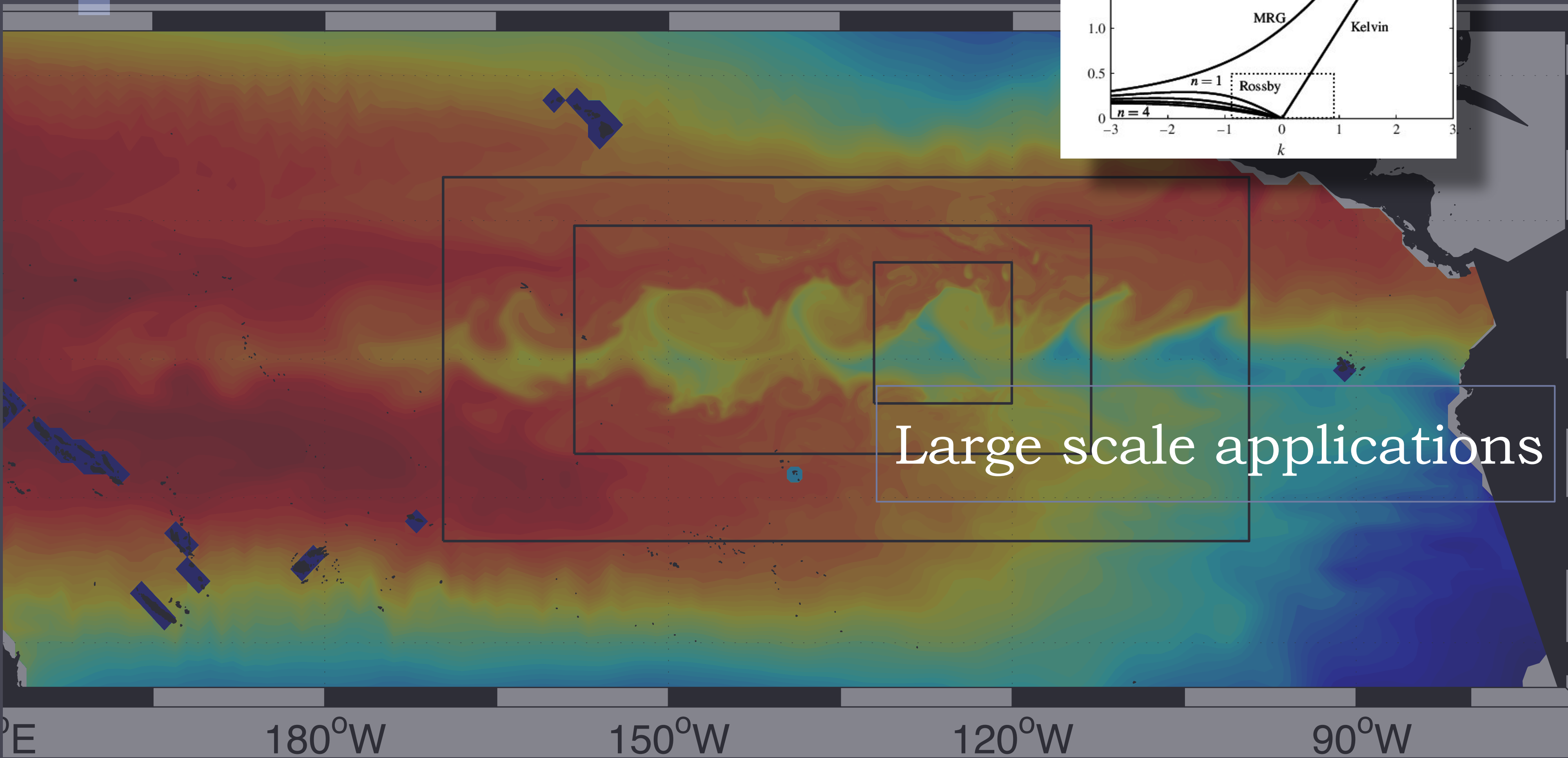
Turbidity patterns (brown) and foam/convergence lines (white)

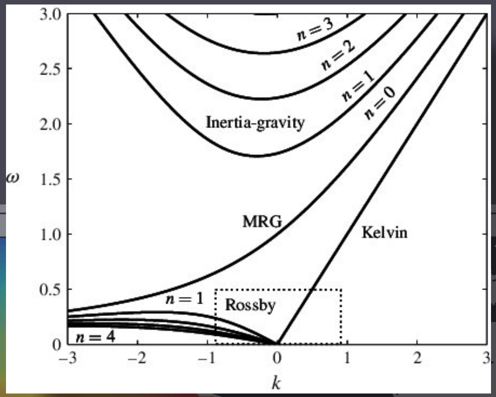
Turbulence cascades
less VLF, more IG eddies





Large scale applications





Quasi-hydrostatic equations

Non-traditional Coriolis terms

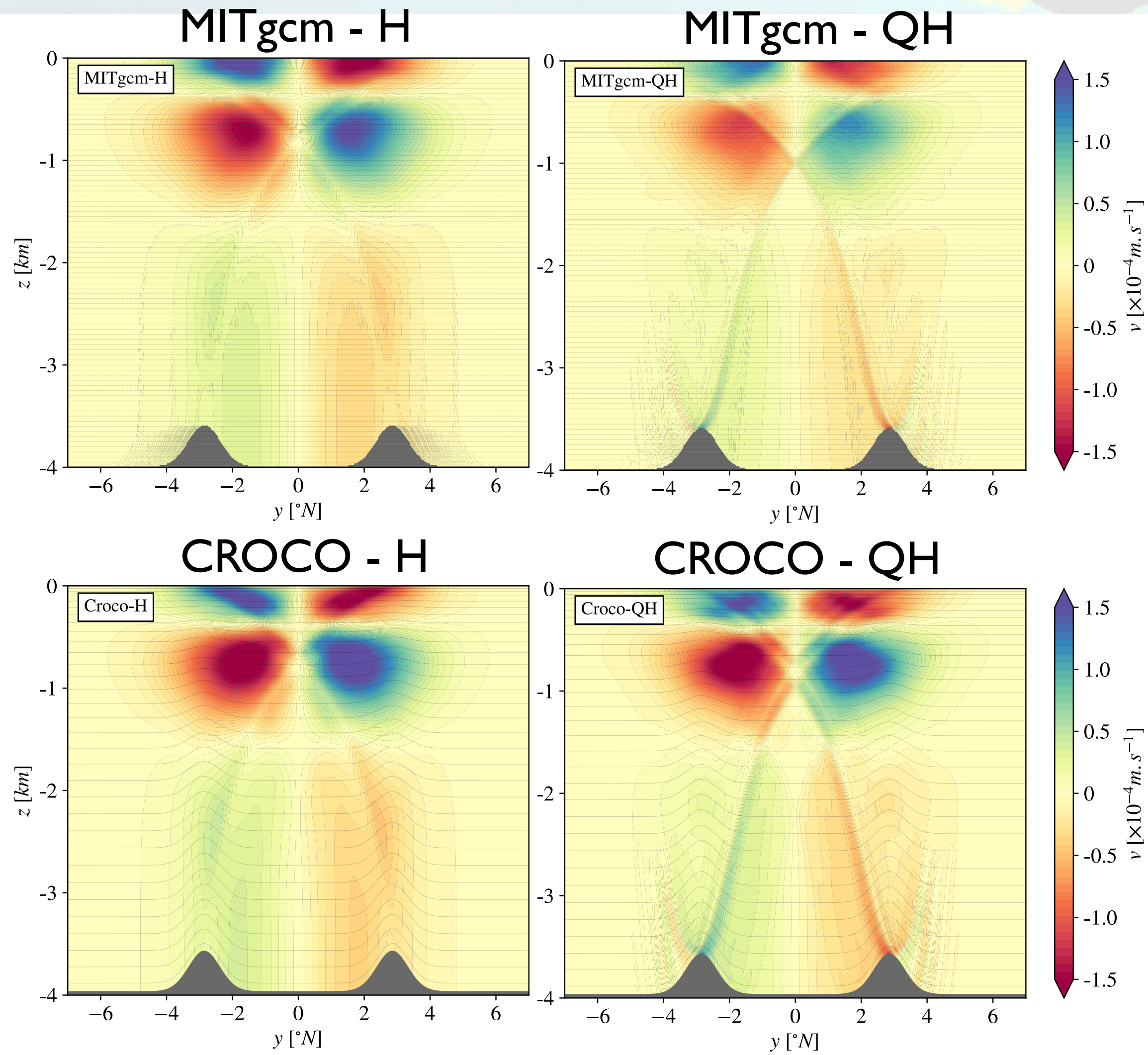
B. Delormes & L. Thomas, Stanford U.

Marshall et al. (1997); Gerkema et al (2008):

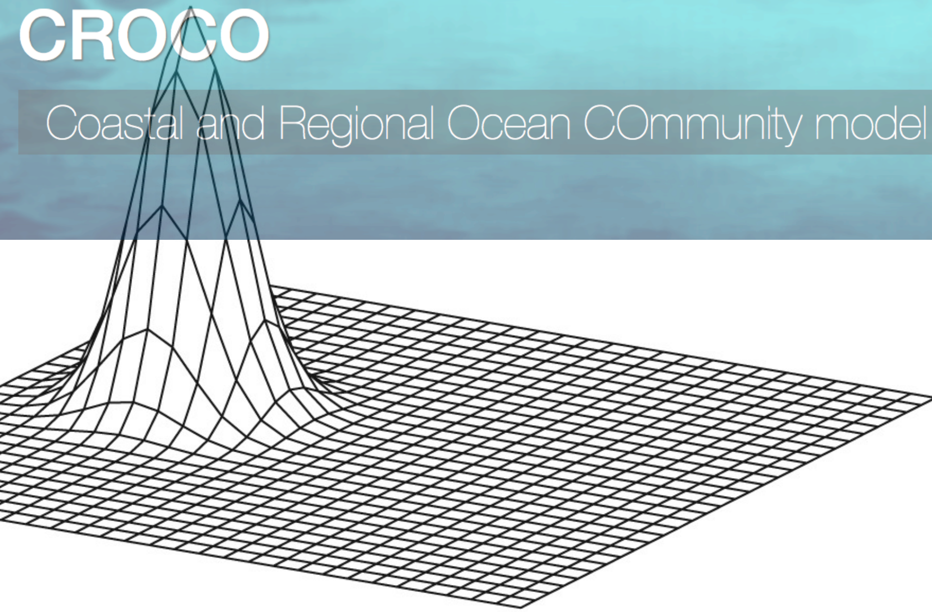
$$\frac{\partial u}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}u\right) - fv + \tilde{f}w = -\frac{\partial \phi}{\partial x} + \mathcal{F}_u + \mathcal{D}_u$$

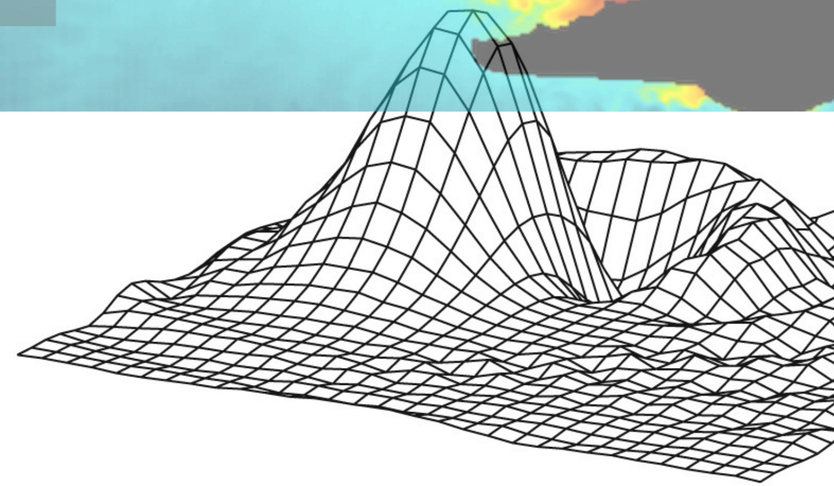
$$\frac{\partial v}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}v\right) + fu = -\frac{\partial \phi}{\partial y} + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial \phi}{\partial z} = -\frac{\rho g}{\rho_0} + \tilde{f}u$$

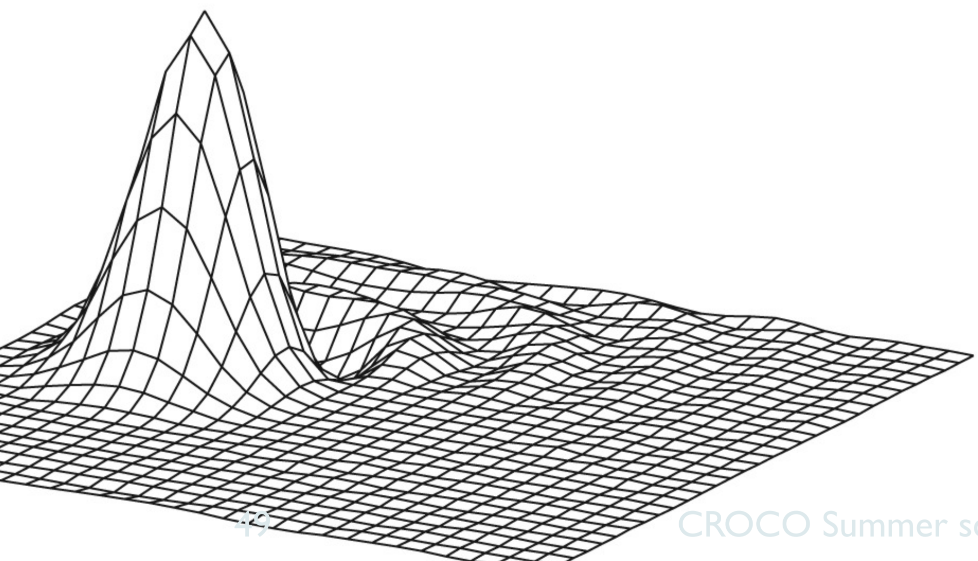


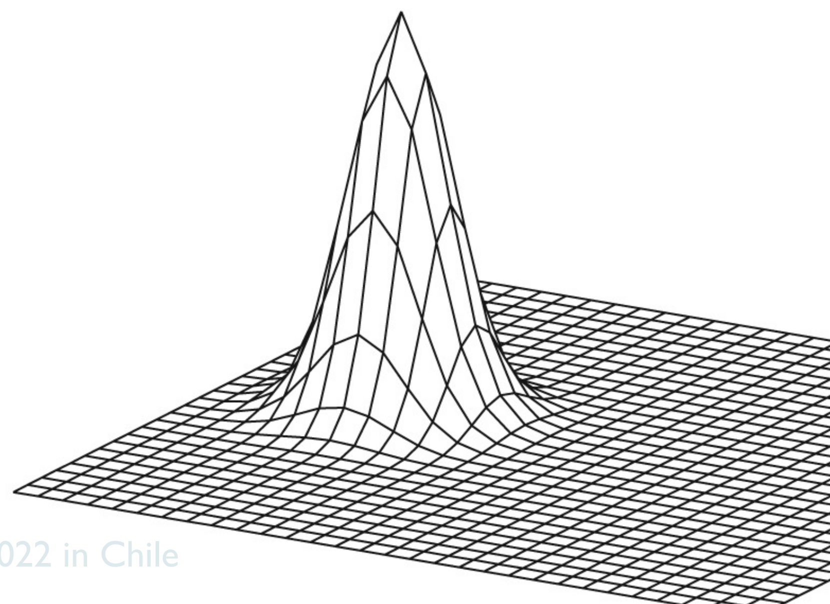
Equatorial wave over topography
150 days into the simulation
wave period = 10 days



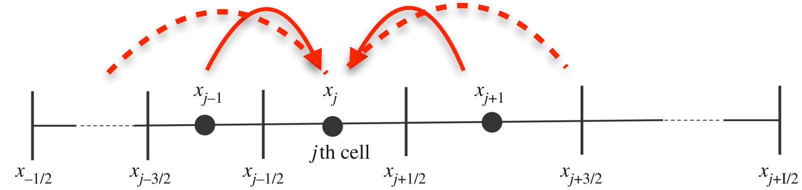


Numerical methods



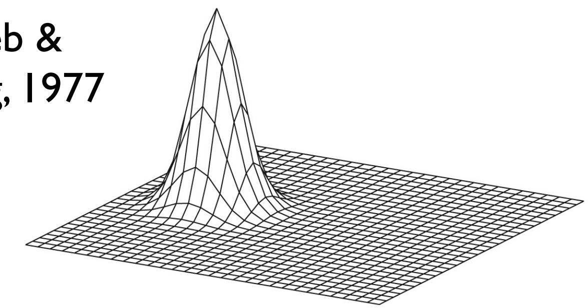


1- High-order benefit



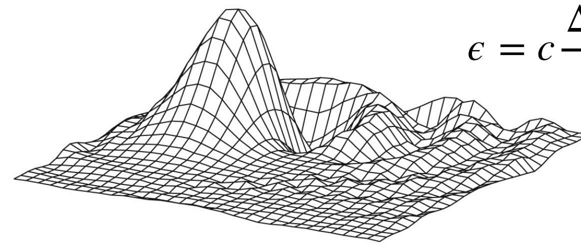
Dispersive centered schemes

Gottlieb & Orszag, 1977



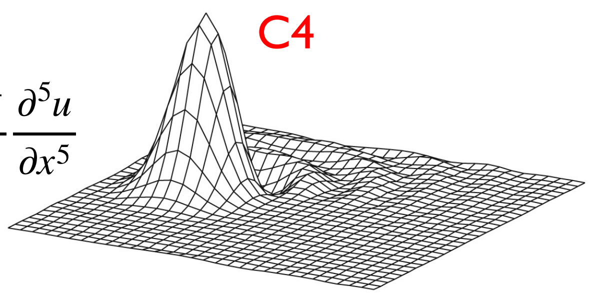
C2

$$\epsilon = c\frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}$$

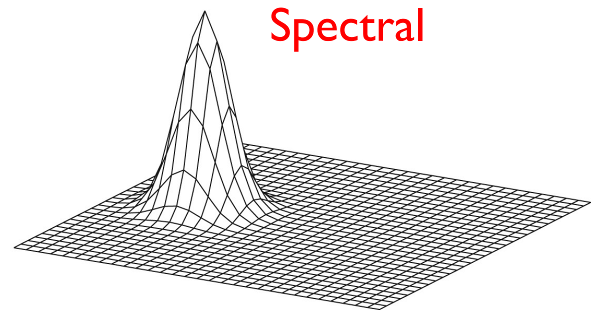


C4

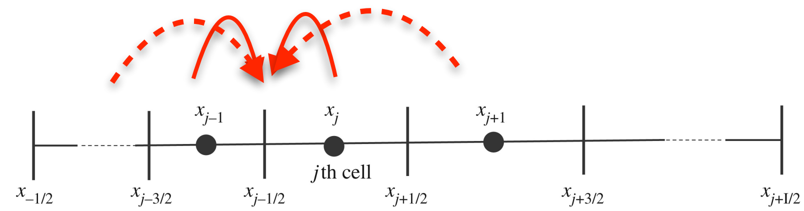
$$\epsilon = c\frac{\Delta x^4}{30}\frac{\partial^5 u}{\partial x^5}$$



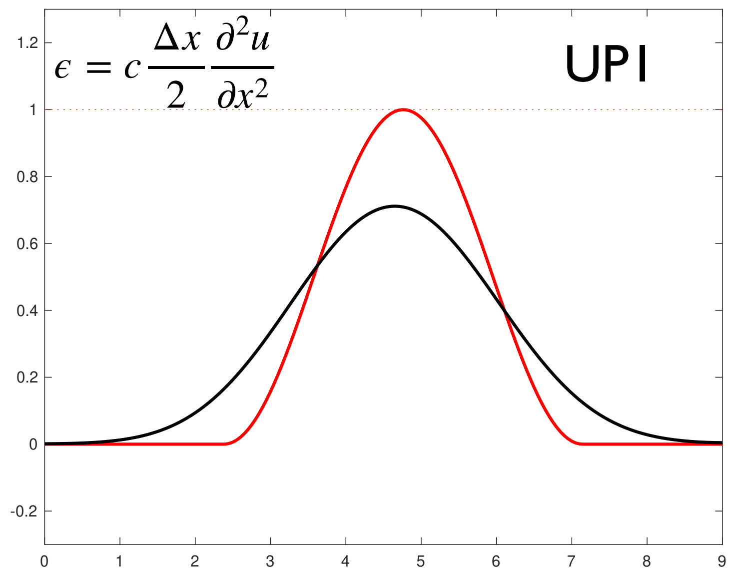
Spectral

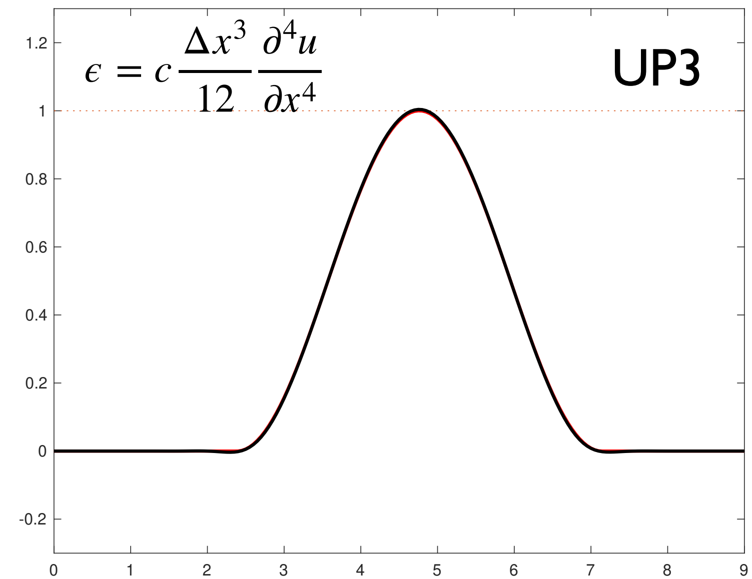


1- High-order benefit



Diffusive upstream schemes





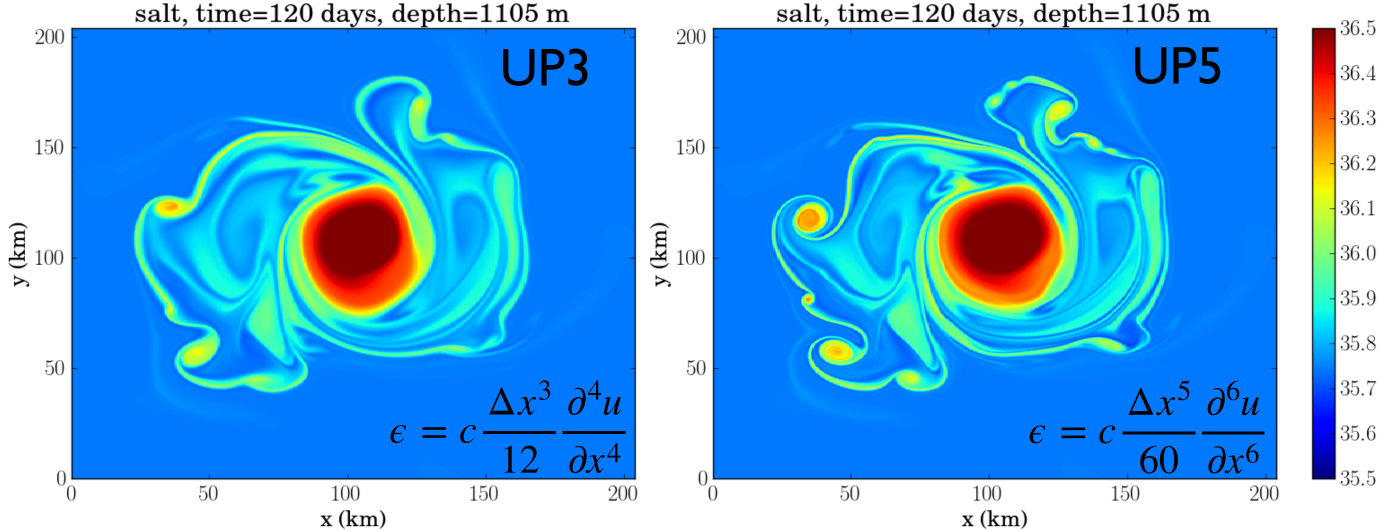
Upwind schemes of any order *n* have optimal damping of dispersion error (Soufflet et al. 2016)

$$\Im(\omega) = -2\left[\frac{c_0 - c_g(k)}{(n+1)\Delta x}\right]$$

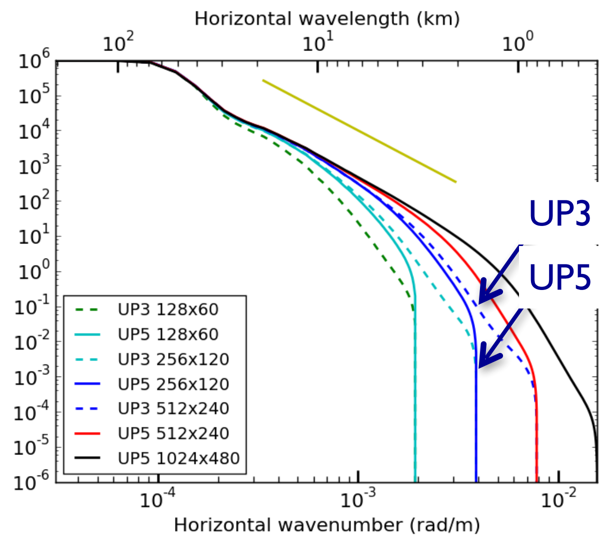
→ Default choice in CROCO

1- High-order benefit: submesoscales

Ideal Meddy (Menesguen et al., 2018)



Effective resolution: x2
Cost: + 6%



1- High-order benefit: Internal solitons

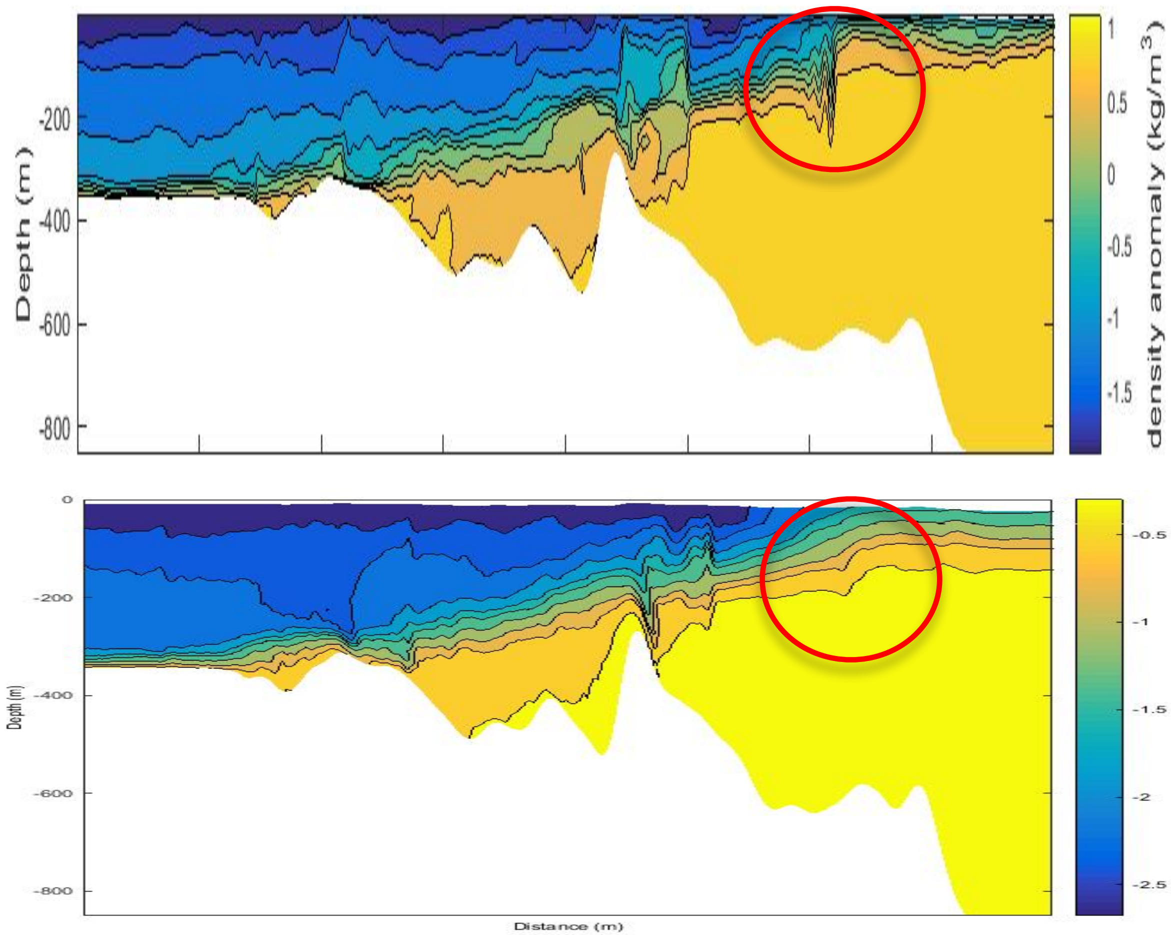
Gibraltar – 200m resolution

CROCO-NBQ

High-order schemes

S-NBQ

Low-order schemes

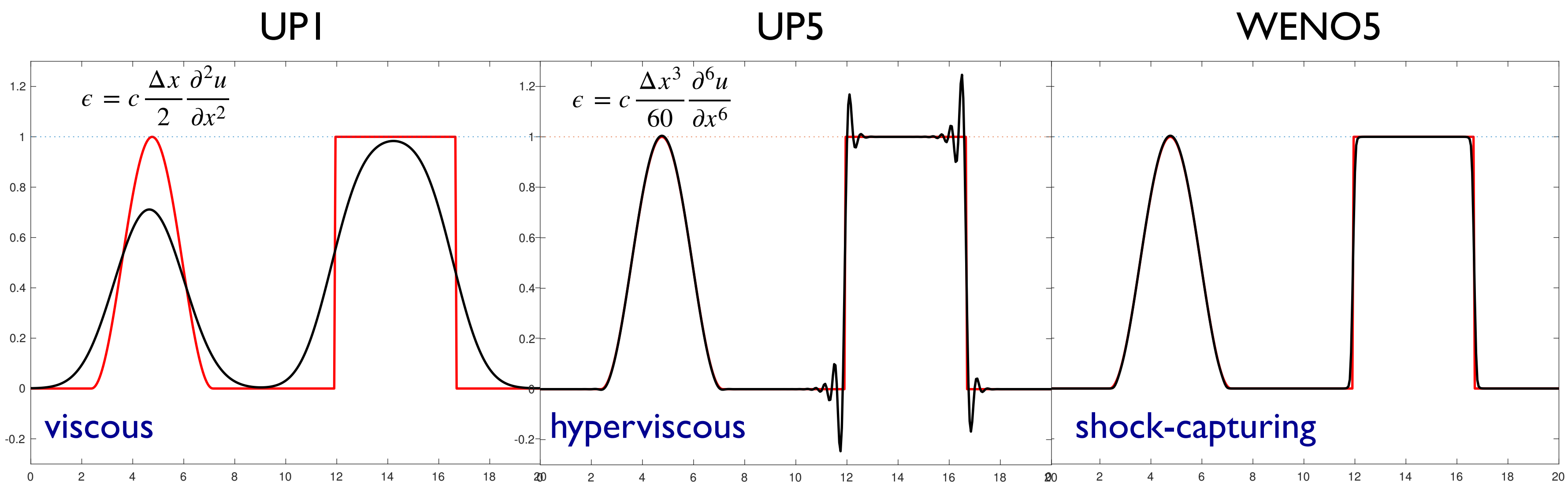


2- Hyperviscous shocks & vortices

Hyperviscosity does not preserve monotonicity

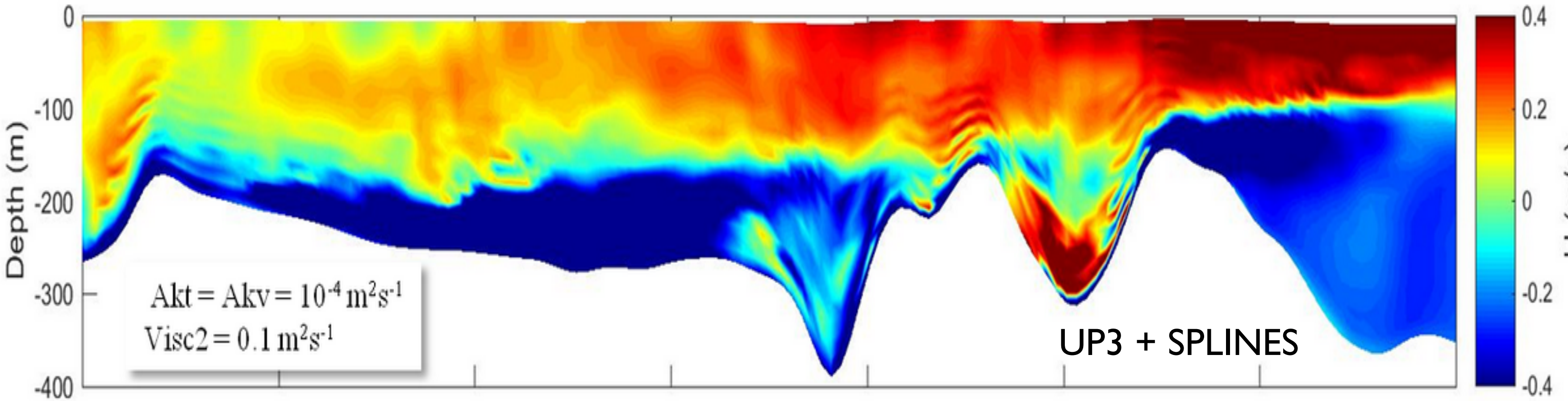
(e.g., hyperdiffusion or hyper-Burger equations) :

→ Oscillations near shocks (Boyd, JSC 1994)
→ Hyperviscous vortices (Jimenez, JFM 1994)

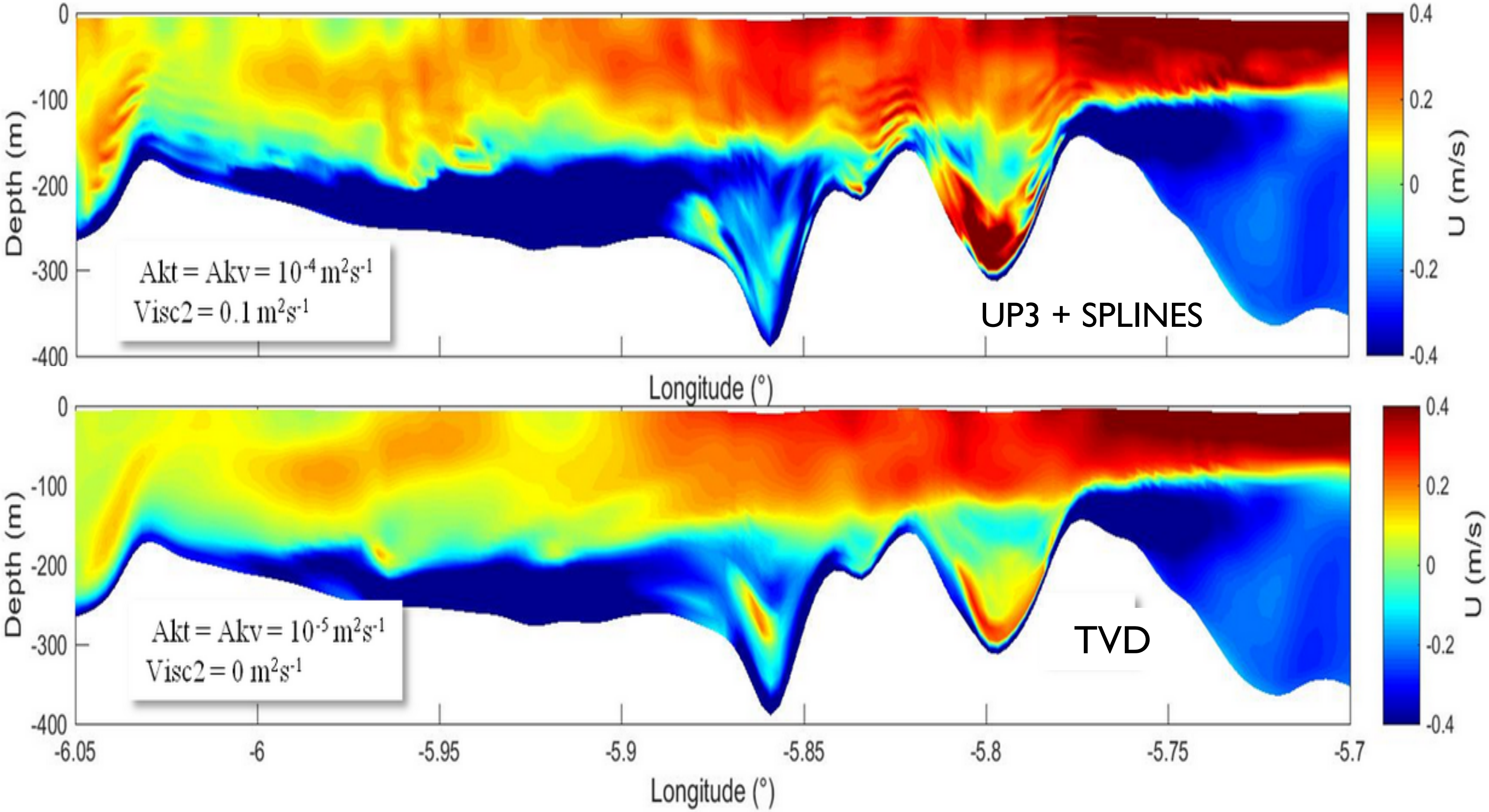


Viscous shock ~ Gibb's shock

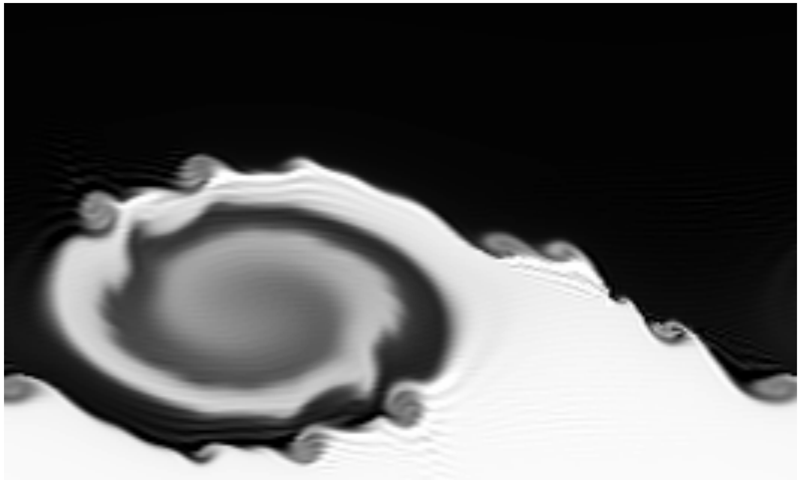
2- Hyperviscous shocks: IGW



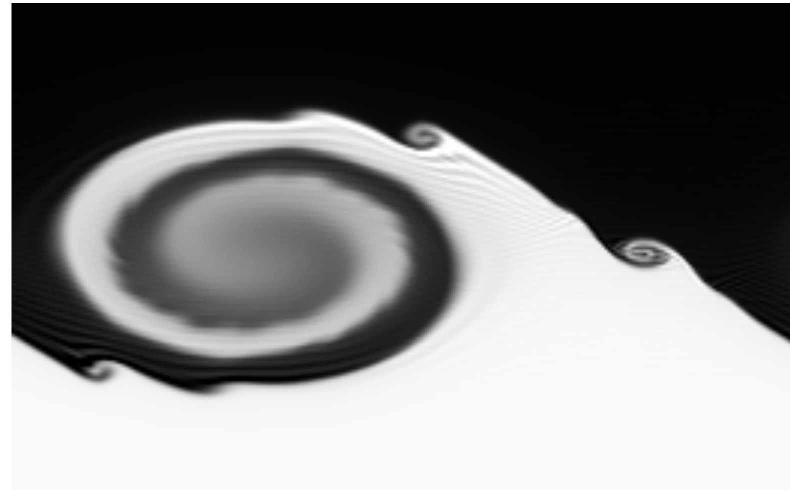
2- Hyperviscous shocks: IGW



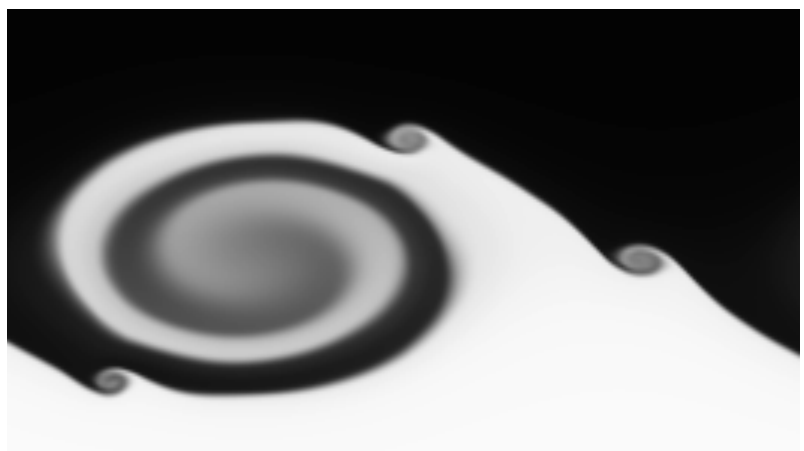
3- HYPERVISCOUS SHOCKS: KHI



Dispersive $[w\rho]_Z$ (AKIMA)



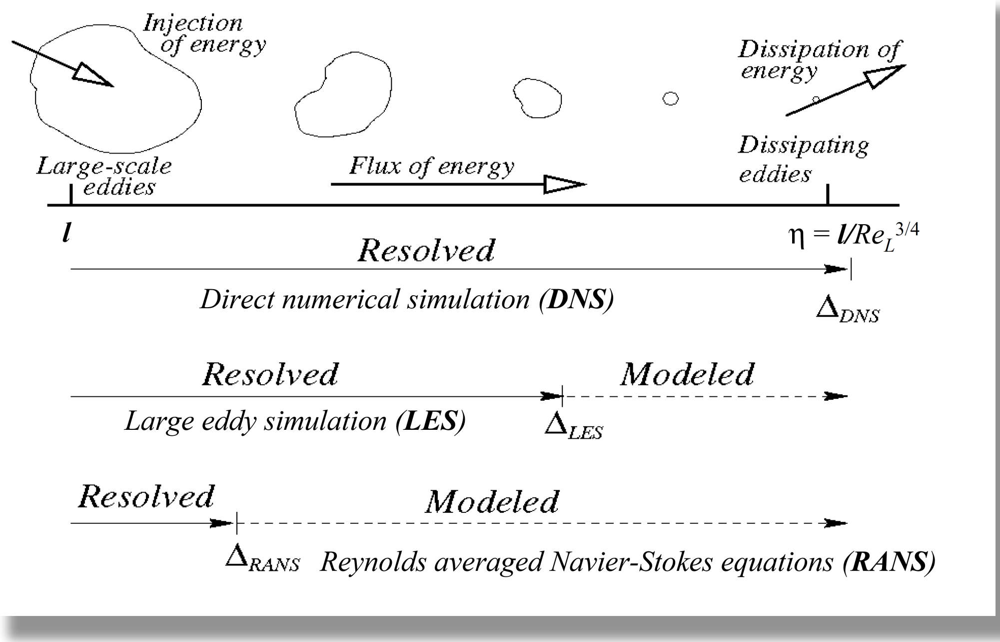
Non monotonic $[w\rho]_Z$ (SPLINES)



All monotonic (WENO5)

Turbulence closure : RANS, LES or MILES

- Turbulent closure (LES / RANS)
  - ✓ 3D GLS (k-epsilon, k-omega …)
  - ✓ 3D Smagorinsky



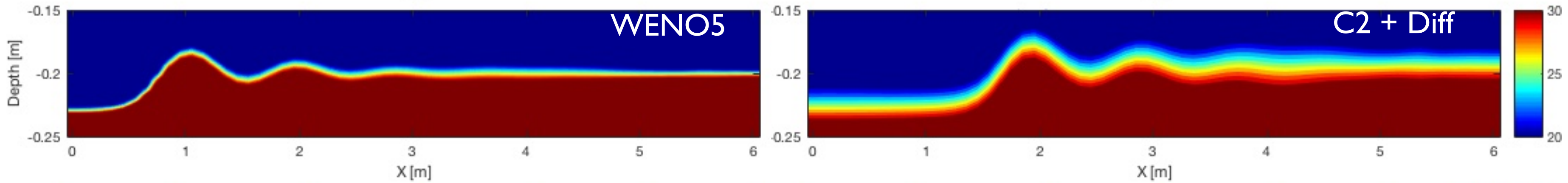
Turbulence closure : RANS, LES or MILES

- Physical / Numerical closure

$$\begin{cases} \nu_{Smag} \sim C_S\, L\, U & C_S \sim 0.01 \\ \nu_{Num} \sim C_N\, L\, U & C_N = 1/12 \leftarrow UP3 \\ & \; 1/60 \leftarrow UP5 \end{cases}$$

(Soufflet et al., 2016)

To be effective, SGS models must be used with high-order advection schemes that include shock-capturing skills (MILES)



CONCLUSIONS

- CROCO is designed for bridging gaps
  - From quasi-geostrophic eddies to micro-turbulence
  - From oceanic to nearshore zones
- CROCO-NBQ is an original approach with many advantages
  - accuracy, performance, versatility
- Multiple tests and applications show good performances and helps further developments
- There is room for improving numerical methods and parametrizations:
  - High-order monotonic advection schemes
  - Immersed boundary conditions
  - Multi-resolution